NCERT Solutions for Class 9 MATHS – Heron's Formula



1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board? Sol. Here, the sides of triangle are a, a and a units.

So, the semi-perimeter of triangle is given by $s = \frac{a+a+a}{2} = \frac{3a}{2}$

Therefore, using Heron's formula, the area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

 $=\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}=\sqrt{\frac{3a}{2}\left(\frac{3a-2a}{2}\right)\left(\frac{3a-2a}{2}\right)\left(\frac{3a-2a}{2}\right)\left(\frac{3a-2a}{2}\right)}$ $=\sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}=\frac{a^2}{4}\sqrt{3}$

Perimeter of equilateral triangle = 3a

According to question, $3a = 180 cm \Rightarrow a = \frac{180}{3} = 60 cm$

Therefore, the area of triangle $=\frac{a^2}{4}\sqrt{3} = \frac{(60)^2}{4}\sqrt{3} = \frac{3600}{4}\sqrt{3} = 900\sqrt{3} \text{ cm}^2$

The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m 2. and 120 m (see Fig.). The advertisements yield an earning of Rs. 5000 m² per year. A company hired one of its walls for 3 months. How much rent did it pay?



Sol. Here, the sides of triangle are a = 122 m, b = 22 m and c = 120m.





So, the semi-perimeter $S = \frac{a+b+c}{2} = \frac{122+22+120}{2} = \frac{264}{2} = 132 \text{ m}$ Therefore, using Heron's formula, the area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{132(132-122)(132-22)(132-120)}$ $= \sqrt{132(10)(110)(12)}$ $= \sqrt{11 \times 12 \times (10)(11 \times 10)(12)}$ $= 11 \times 12 \times 10 = 1320 \text{ m}^2$ Earning by advertisement in 1 year (12 months) = Rs. 5000 × 1320 \Rightarrow Earning by advertisement in 1 month $= \frac{\text{Rs. } 5000 \times 1320}{12}$ \Rightarrow Earning by advertisement in 3 month $= \frac{\text{Rs. } 5000 \times 1320}{12} \times 3 = \text{Rs. } 16,50,000$

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEPTHE PARK GREEN AND CLEAN" (see Fig.). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

Sol. Here, the sides of triangle are a=15 m, b=11m and c=6m. So, the semi-perimeter of triangle is given by $s = \frac{a+b+c}{2} = \frac{15+11+6}{2} = \frac{32}{2} = 16m$ Therefore, using Heron's formula, the area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{16(16-15)(16-11)(16-6)}$ $= \sqrt{16(1)(5)(10)}$

$$= \sqrt{4 \times 4 \times (1)(5)(5 \times 2)}$$

$$=4 \times 5\sqrt{2} = 20\sqrt{2} \text{ m}^2$$

Hence, the area painted in colour is $20\sqrt{2}$ m².





4. Find the area of a triangle whose two sides are 18cm and 10cm and the perimeter is 42cm.

Sol. Here, the sides of triangle are a = 18 cm, b = 10 cm and perimeter is 42 cm.

We know that the perimeter of triangle = a + b + c

$$\Rightarrow$$
 42 = 18 + 10 + c

$$\Rightarrow c = 14 cm$$

So, the semi-perimeter of triangle is given by

$$s = \frac{a+b+c}{2} = \frac{42}{2} = 21 \ cm$$

Therefore, using Heron's formula, the area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21(21-18)(21-10)(21-14)}$$

= $\sqrt{21(3)(11)(7)}$
= $\sqrt{7 \times 3 \times (3)(11)(7)}$
= $7 \times 3\sqrt{11} = 21\sqrt{11} \text{ cm}^2$

Hence, the area of triangle is $21\sqrt{11}$ cm².

5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.

Sol. Perimeter of triangle = 540 cm

The ratio of sides of triangle = 12: 17: 25

Let, one of the sides of triangle a = 12x

Therefore, remaining two sides are b = 17x and c = 25x.

We know that the perimeter of triangle = a + b + c

$$\Rightarrow 540 = 12x + 17x + 25x$$
$$\Rightarrow 540 = 54x$$

$$\Rightarrow$$
540=54x

$$\Rightarrow x = \frac{540}{54} = 10$$

So, the sides of triangle are $a=12\times10=120$ cm, $b=17\times10=170$ cm and $c=25\times10=250$ cm.

So, the semi-perimeter of triangle is given by

$$s = \frac{a+b+c}{2} = \frac{540}{2} = 270 \, cm$$

Therefore, using Heron's formula, the area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

 $=\sqrt{270(270-120)(270-170)(270-250)}$

$$=\sqrt{270(150)(100)(20)}$$

 $=9000 \, \text{cm}^2$

Hence, the area of triangle is $=9000 \text{ cm}^2$.

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6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle. Sol. Perimeter of triangle = 30 cm

Two sides of triangle b = 12 cm and c = 12 cm.

Let, the third side $= a \ cm$

We know that the perimeter of triangle = a + b + c

 $\Rightarrow \qquad 30 = a + 12 + 12$ $\Rightarrow \qquad 30 - 24 = a$ $\Rightarrow \qquad a = 6$

So, the semi-perimeter of triangle is given by

$$s = \frac{a+b+c}{2} = \frac{30}{2} = 15$$
 cm

Therefore, using Heron's formula, the area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{15(15-6)(15-12)(15-12)}$$
$$= \sqrt{15(9)(3)(3)}$$
$$= 9\sqrt{15} \text{ cm}^2$$

Hence, the area of triangle is $=9\sqrt{15}$ cm².

7. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Sol. Join the diagonal AC of quadrilateral ABC.

Here, the sides of triangle ABC are a = 30 m, b = 30 m and c = 48 m. So, the semi-perimeter of triangle

$$s = \frac{a+b+c}{2} = \frac{30+30+48}{2} = \frac{108}{2} = 54 \,\mathrm{m}$$

Therefore, area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

 $=\sqrt{54(54-30)(54-30)(54-48)}=\sqrt{54(24)(24)(6)}=\sqrt{186624}=432\,\mathrm{m}^2$

Hence, area of quadrilateral $= 2 \times 432 = 864 \text{ m}^2$

Therefore, the area grazed by each cow
$$=\frac{\text{Total area}}{\text{Number of cows}}=\frac{864}{18}=48 \text{ m}^2$$



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- 8. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig.), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?
- Sol. Here, the sides of triangle are a = 20 cm, b = 50 cm and c = 50 cm. So, the semi-perimeter of triangle $S = \frac{a+b+c}{2} = \frac{20+50+50}{2} = \frac{120}{2} = 60 cm$ Therefore, using Heron's formula, area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{60(60-20(60-50)(60-50)} = \sqrt{60(40)(10)(10)}$ = $200\sqrt{6}$ cm² So, area of 10 triangular pieces of cloths = $10 \times 200\sqrt{6} = 2000\sqrt{6}$ cm² Hence, the area of cloths of each colour = $\frac{2000\sqrt{6}}{2} = 1000\sqrt{6}$ cm²



9. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig.). Find the cost of polishing the tiles at the rate of 50p per cm².



Sol. Here, the sides of triangle are a = 9 cm, b = 28 cm and c = 35 cm. So, the semi-perimeter of triangle $s = \frac{a+b+c}{2} = \frac{9+28+35}{2} = \frac{72}{2} = 36 cm$ Therefore, using Heron's formula area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{36(36-9)(36-28)(36-35)} = \sqrt{36(27)(8)(1)}$ $= \sqrt{7776}$

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 $= 88.2 \text{ cm}^2$ (approx.)

So, area of each triangular tile $= 88.2 \text{ cm}^2$

Therefore, area of each triangular 16 tiles $=16 \times 88.2 = 1411.2 \text{ cm}^2$

Hence, the cost of polishing the tiles at the rate of 50p per cm² = $Rs.0.50 \times 1411.2 = Rs.705.60$

- **10.** A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.
- **Sol.** Draw CF || AD and $CG \perp AB$.



 $=\sqrt{21(8)(7)(6)}$ $=\sqrt{7056}$

$$= 84 m^2$$

But, the area of triangle $BCF = \frac{1}{2} \times BF \times CG$

So,
$$\frac{1}{2} \times BF \times CG = 84$$





$$\Rightarrow \frac{1}{2} \times 15 \times CG = 84$$
$$\Rightarrow CG = \frac{84 \times 2}{15} = 11.2 m$$

Therefore, area of trapezium $ABCD = \frac{1}{2} \times (AB + CD) \times CG$

$$= \frac{1}{2} \times (25 + 10) \times 11.2$$

= 35 × 5.6
= 196 m²

Hence, the area of the field $= 196 \text{ m}^2$.

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