## NTSE

NCERT Solutions for Class 9
MATHS - Heron's Formula

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1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side ' $a$ '. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm , what will be the area of the signal board?
Sol. Here, the sides of triangle are $a, a$ and $a$ units.
So, the semi-perimeter of triangle is given by $s=\frac{a+a+a}{2}=\frac{3 a}{2}$
Therefore, using Heron's formula, the area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{\frac{3 a}{2}\left(\frac{3 a}{2}-a\right)\left(\frac{3 a}{2}-a\right)\left(\frac{3 a}{2}-a\right)}=\sqrt{\frac{3 a}{2}\left(\frac{3 a-2 a}{2}\right)\left(\frac{3 a-2 a}{2}\right)\left(\frac{3 a-2 a}{2}\right)}$
$=\sqrt{\frac{3 a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}=\frac{a^{2}}{4} \sqrt{3}$
Perimeter of equilateral triangle $=3 a$
According to question, $3 a=180 \mathrm{~cm} \Rightarrow a=\frac{180}{3}=60 \mathrm{~cm}$
Therefore, the area of triangle $=\frac{a^{2}}{4} \sqrt{3}=\frac{(60)^{2}}{4} \sqrt{3}=\frac{3600}{4} \sqrt{3}=900 \sqrt{3} \mathrm{~cm}^{2}$
2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are $122 \mathrm{~m}, 22 \mathrm{~m}$ and 120 m (see Fig.). The advertisements yield an earning of Rs. $5000 \mathrm{~m}^{2}$ per year. A company hired one of its walls for 3 months. How much rent did it pay?


Sol. Here, the sides of triangle are $a=122 m, b=22 m$ and $c=120 m$.

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So, the semi-perimeter $S=\frac{a+b+c}{2}=\frac{122+22+120}{2}=\frac{264}{2}=132 \mathrm{~m}$
Therefore, using Heron's formula, the area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{132(132-122)(132-22)(132-120)}$
$=\sqrt{132(10)(110)(12)}$
$=\sqrt{11 \times 12 \times(10)(11 \times 10)(12)}$
$=11 \times 12 \times 10=1320 \mathrm{~m}^{2}$
Earning by advertisement in 1 year ( 12 months $)=$ Rs. $5000 \times 1320$
$\Rightarrow$ Earning by advertisement in 1 month $=\frac{\text { Rs. } 5000 \times 1320}{12}$
$\Rightarrow$ Earning by advertisement in 3 month $=\frac{\text { Rs. } 5000 \times 1320}{12} \times 3=$ Rs. $16,50,000$
3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEPTHE PARK GREEN AND CLEAN" (see Fig.). If the sides of the wall are $15 \mathrm{~m}, 11 \mathrm{~m}$ and 6 m , find the area painted in colour.


15 m
Sol. Here, the sides of triangle are $a=15 m, b=11 m$ and $c=6 m$.
So, the semi-perimeter of triangle is given by
$s=\frac{a+b+c}{2}=\frac{15+11+6}{2}=\frac{32}{2}=16 \mathrm{~m}$
Therefore, using Heron's formula, the area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{16(16-15)(16-11)(16-6)}$
$=\sqrt{16(1)(5)(10)}$
$=\sqrt{4 \times 4 \times(1)(5)(5 \times 2)}$
$=4 \times 5 \sqrt{2}=20 \sqrt{2} \mathrm{~m}^{2}$
Hence, the area painted in colour is $20 \sqrt{2} \mathrm{~m}^{2}$.

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4. Find the area of a triangle whose two sides are 18 cm and 10 cm and the perimeter is 42 cm .

Sol. Here, the sides of triangle are $a=18 \mathrm{~cm}, b=10 \mathrm{~cm}$ and perimeter is 42 cm .
We know that the perimeter of triangle $=a+b+c$
$\Rightarrow 42=18+10+c$
$\Rightarrow c=14 \mathrm{~cm}$
So, the semi-perimeter of triangle is given by

$$
s=\frac{a+b+c}{2}=\frac{42}{2}=21 \mathrm{~cm}
$$

Therefore, using Heron's formula, the area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{21(21-18)(21-10)(21-14)}$
$=\sqrt{21(3)(11)(7)}$
$=\sqrt{7 \times 3 \times(3)(11)(7)}$
$=7 \times 3 \sqrt{11}=21 \sqrt{11} \mathrm{~cm}^{2}$
Hence, the area of triangle is $21 \sqrt{11} \mathrm{~cm}^{2}$.
5. Sides of a triangle are in the ratio of $12: 17: 25$ and its perimeter is 540 cm . Find its area.

Sol. Perimeter of triangle $=540 \mathrm{~cm}$
The ratio of sides of triangle $=12: 17: 25$
Let, one of the sides of triangle $a=12 x$
Therefore, remaining two sides are $b=17 x$ and $c=25 x$.
We know that the perimeter of triangle $=a+b+c$
$\Rightarrow 540=12 x+17 x+25 x$
$\Rightarrow 540=54 x$
$\Rightarrow x=\frac{540}{54}=10$
So, the sides of triangle are $a=12 \times 10=120 \mathrm{~cm}, \mathrm{~b}=17 \times 10=170 \mathrm{~cm}$ and $c=25 \times 10=250 \mathrm{~cm}$.
So, the semi-perimeter of triangle is given by

$$
s=\frac{a+b+c}{2}=\frac{540}{2}=270 \mathrm{~cm}
$$

Therefore, using Heron's formula, the area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{270(270-120)(270-170)(270-250)}$
$=\sqrt{270(150)(100)(20)}$
$=9000 \mathrm{~cm}^{2}$
Hence, the area of triangle is $=9000 \mathrm{~cm}^{2}$.

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6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm . Find the area of the triangle.

Sol. Perimeter of triangle $=30 \mathrm{~cm}$
Two sides of triangle $\mathrm{b}=12 \mathrm{~cm}$ and $\mathrm{c}=12 \mathrm{~cm}$.
Let, the third side $=a \mathrm{~cm}$
We know that the perimeter of triangle $=a+b+c$
$\Rightarrow \quad 30=a+12+12$
$\Rightarrow \quad 30-24=a$
$\Rightarrow \quad a=6$
So, the semi-perimeter of triangle is given by

$$
s=\frac{a+b+c}{2}=\frac{30}{2}=15 \mathrm{~cm}
$$

Therefore, using Heron's formula, the area of triangle $=\sqrt{s(s-a)(\mathrm{s}-\mathrm{b})(\mathrm{s}-\mathrm{c})}$
$=\sqrt{15(15-6)(15-12)(15-12)}$
$=\sqrt{15(9)(3)(3)}$
$=9 \sqrt{15} \mathrm{~cm}^{2}$
Hence, the area of triangle is $=9 \sqrt{15} \mathrm{~cm}^{2}$.
7. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m , how much area of grass field will each cow be getting?
Sol. Join the diagonal AC of quadrilateral ABC .
Here, the sides of triangle ABC are $\mathrm{a}=30 \mathrm{~m}, \mathrm{~b}=30 \mathrm{~m}$ and $\mathrm{c}=48 \mathrm{~m}$.
So, the semi-perimeter of triangle

$$
\mathrm{s}=\frac{a+b+c}{2}=\frac{30+30+48}{2}=\frac{108}{2}=54 \mathrm{~m}
$$

Therefore, area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

$$
=\sqrt{54(54-30)(54-30)(54-48)}=\sqrt{54(24)(24)(6)}=\sqrt{186624}=432 \mathrm{~m}^{2}
$$

Hence, area of quadrilateral $=2 \times 432=864 \mathrm{~m}^{2}$
Therefore, the area grazed by each cow $=\frac{\text { Total area }}{\text { Number of cows }}=\frac{864}{18}=48 \mathrm{~m}^{2}$


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8. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig.), each piece measuring $20 \mathrm{~cm}, 50 \mathrm{~cm}$ and 50 cm . How much cloth of each colour is required for the umbrella?


Therefore, using Heron's formula, area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{60(60-20(60-50)(60-50)}=\sqrt{60(40)(10)(10)}$
$=200 \sqrt{6} \mathrm{~cm}^{2}$
So, area of 10 triangular pieces of cloths $=10 \times 200 \sqrt{6}=2000 \sqrt{6} \mathrm{~cm}^{2}$
Hence, the area of cloths of each colour $=\frac{2000 \sqrt{6}}{2}=1000 \sqrt{6} \mathrm{~cm}^{2}$
9. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being $9 \mathrm{~cm}, 28 \mathrm{~cm}$ and 35 cm (see Fig.). Find the cost of polishing the tiles at the rate of 50 p per $\mathrm{cm}^{2}$.


Sol. Here, the sides of triangle are $\mathrm{a}=9 \mathrm{~cm}, \mathrm{~b}=28 \mathrm{~cm}$ and $\mathrm{c}=35 \mathrm{~cm}$.
So, the semi-perimeter of triangle $s=\frac{a+b+c}{2}=\frac{9+28+35}{2}=\frac{72}{2}=36 \mathrm{~cm}$
Therefore, using Heron's formula area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{36(36-9)(36-28)(36-35)}=\sqrt{36(27)(8)(1)}$
$=\sqrt{7776}$

$$
=88.2 \mathrm{~cm}^{2} \text { (approx.) }
$$

So, area of each triangular tile $=88.2 \mathrm{~cm}^{2}$
Therefore, area of each triangular 16 tiles $=16 \times 88.2=1411.2 \mathrm{~cm}^{2}$
Hence, the cost of polishing the tiles at the rate of 50 p per $\mathrm{cm}^{2}=$ Rs. $0.50 \times 1411.2=$ Rs. 705.60
10. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m . The non-parallel sides are 14 m and 13 m . Find the area of the field.
Sol. Draw $\mathrm{CF} \| \mathrm{AD}$ and $C G \perp A B$.


In quadrilateral ADCF ,
$C F \| A D$
[ $\because$ By construction]
$C D \| A F$
$[\because \mathrm{ABCD}$ is a trapezium $]$
Therefore, ADCF is a parallelogram. So, $A D=C F=13 \mathrm{~m}$ and $C D=A F=10 \mathrm{~m}$

$$
\text { [ } \because \text { Opposite sides of a parallelogram] }
$$

Therefore, $B F=A B-A F=25-10=15 \mathrm{~m}$
Here, the sides of triangle are $\mathrm{a}=13 \mathrm{~m}, \mathrm{~b}=14 \mathrm{~m}$ and $\mathrm{c}=15 \mathrm{~m}$.
So, the semi-perimeter of triangle

$$
s=\frac{a+b+c}{2}=\frac{13+14+15}{2}=\frac{42}{2}=21 \mathrm{~m}
$$

Therefore, using Heron's formula, area of triangle $B C F=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{21(21-13)(21-14)(21-15)}$
$=\sqrt{21(8)(7)(6)}$
$=\sqrt{7056}$
$=84 \mathrm{~m}^{2}$
But, the area of triangle $B C F=\frac{1}{2} \times B F \times C G$
So, $\frac{1}{2} \times B F \times C G=84$

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$\Rightarrow \frac{1}{2} \times 15 \times C G=84$
$\Rightarrow C G=\frac{84 \times 2}{15}=11.2 \mathrm{~m}$
Therefore, area of trapezium $A B C D=\frac{1}{2} \times(A B+C D) \times C G$
$=\frac{1}{2} \times(25+10) \times 11.2$
$=35 \times 5.6$
$=196 \mathrm{~m}^{2}$
Hence, the area of the field $=196 \mathrm{~m}^{2}$.

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