1. Which of the following statements are true and which are false? Give reasons for your answers.
(i) Only one line can pass through a single point.
(ii) There are an infinite number of lines which pass through two distinct points.
(iii) A terminated line can be produced indefinitely on both the sides.
(iv) If two circles are equal, then their radii are equal.
(v) In Fig. if AB = PQ and PQ = XY, then AB = XY

Sol. (i) False, as there are infinite number of line that can pass through one point.
(ii) False, because one and only one line can be drawn through two distinct points.
(iii) True, because a terminated line can be produced both the sides infinitely.
(iv) True, if two circles are equal (i.e. their areas πr^2 are equal), then their radii are also equal.
(v) True, according to Euclid's axiom, "Things which are equal to the same thing are equal to one another".

2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?
(i) Parallel lines (ii) perpendicular lines (iii) line segment (iv) Radius of a circle
(v) square

Sol.

Yes there are other terms which need to be defined first, they are:
Plane: flat surfaces in which geometric figures can be drawn are known are plane.
Point: A dimensionless dot which is drawn on a plane surface is known as point.
Line: A collection of points that has only length and no breadth is known as a line. And it can be extended on both directions.
A: Parallel lines- Parallel lines are lines in which never intersect each other and are always at a perpendicular distance between them which is constant distance. Parallel lines can be two or more lines.
B: Perpendicular lines- Perpendicular lines are lines which intersect each other in a plane at right angles then the lines are said to be perpendicular to each other.
C: Line Segment- When a line cannot be extended any further because of its two end points then the line is known as a line segment.
D: Radius of circle- A radius of a circle can be a line from any point on the circumference to the centre of the circle.
E: Square- A quadrilateral in which all the four sides are said to be equal and each of its internal angle is right angles.

3. Consider two 'postulates' given below:
(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.
(ii) There exist at least three points that are not on the same line.
Do these postulates contain any undefined terms? Are these postulates consistent?
Do they follow from Euclid's postulates? Explain.

Sol. There are several undefined terms which should be listed. They are consistent, because they deal with two different situations —
(i) says that
Given two points A and B, there is a point C lying on the line in between them;
(ii) says that:
Given A and B, you can take C not lying on the line through A and B.
These 'postulates' do not follow from Euclid's postulates. However, they follow from Axiom 5.1.

4. If a point C lies between two points A and B such that AC = BC, then prove that $AC = \frac{1}{2}AB$. Explain by drawing the figure.

Sol.

Given: AC = BC

$\Rightarrow AC + AC = AC + BC$ [$\because$ Equals are added to equals]

$\Rightarrow 2AC = AB$ [$\because$ AC + BC coincides with AB]

$\Rightarrow AC = \frac{1}{2}AB$ [$\because$ Things which are halves of the same things are equal to one another]

5. In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

Sol. Let C and D are two mid-points of line segment AB. According to question 4, we have, $AC = \frac{1}{2}AB$ and

$AD = \frac{1}{2}AB$

$\Rightarrow AC = AD$ [$\because$ Things which are equal to the same thing are equal to one another]

It is possible only if C and D coincide with each other.
Hence, the mid-point C is unique.

6. In Figure, if AC = BD, then prove that AB = CD.

Sol. Given that: AC = BD

$\Rightarrow AC - BC = BD - BC$ [$\because$ If equals are subtracted from equals, the remainders are equal]

$\Rightarrow AB = CD$

7. Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate.)

Sol. The whole is greater than the part.

Since this is true for anything in any part of the world, this is a universal truth.

8. How would you rewrite Euclid's fifth postulate so that it would be easier to understand?

Sol. If two lines intersect the third line in such a way that the sum of the interior angles is less than 180°, then the two lines intersect each other.

9. Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

Sol. Yes, Euclid's fifth postulate implies the existence of parallel lines. Because if the two lines intersect the third line in such a way that the sum of the interior angles is less than 180°, then the two lines intersect each other but if the sum of interior angles is 180°, then the lines don't intersect or lines are parallel.