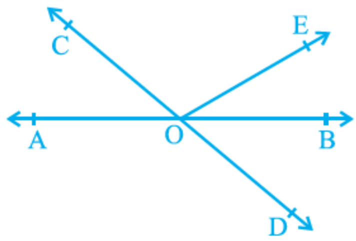
1. In Figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.



Sol. Lines AB and CD intersect at O

$\therefore$ $\angle AOC = \angle BOD$ [Vertically Opposite Angles]

But $\angle BOD = 40°$ ………(1) [Given]

$\therefore$ $\angle AOC = 40°$ ………(2)

Now, $\angle AOC + \angle BOE = 70°$

$\Rightarrow$ $40° + \angle BOE = 70°$ [Using (2) $\Rightarrow$ $\angle BOE = 70° - 40°$]

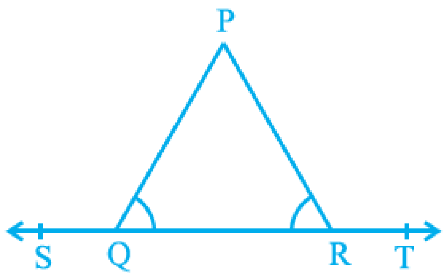
$\Rightarrow$ $\angle BOE = 30°$

Again, reflex $\angle COE = \angle COD + \angle BOD + \angle BOE$

$= \angle COD + 40° + 30°$ ……..(3) [Using (1) and (2)]

$= 180° + 40° + 30° = 250°$

2. In figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Sol. Ray QP stands on line ST

$\therefore$ $\angle PQS + \angle PQR = 180°$ ……(1)

Ray RP stands on line ST

$\therefore$ $\angle PRQ + \angle PRT = 180°$ …….(2)

[Linear Pair Axiom]

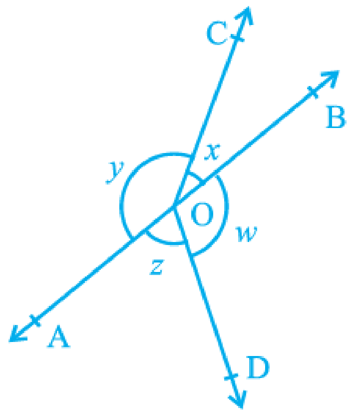
From (1) and (2), we obtain

$\angle PQS + \angle PQR = \angle PRQ + \angle PRT$

$\Rightarrow$ $\angle PQS = \angle PRT$

[Since, $\angle PRQ = \angle$ PQR given]

3. In Figure, if $x + y = w + z$, then prove that AOB is a line.



Sol. $x + y = w + z$ ....(1)

[Given $\therefore$ The sum of all the angles round a point is equal to $360°$]

$\therefore$ $x + y + w + z = 360°$

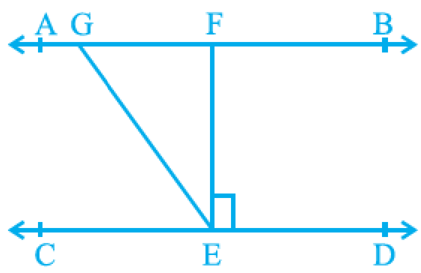
$\Rightarrow$ $x + y + x + y = 360°$ [Using (1)]

$\Rightarrow$ $2(x + y) = 360°$

$\Rightarrow$ $x + y = 180°$

$\therefore$ AOB is a line.

4. In Figure, if $AB \| CD, EF \perp CD$ and $\angle GED = 126°$, find $\angle AGE, \angle GEF$ and $\angle FGE$.



Sol. **(i)** $\angle AGE = \angle GED = 126°$ **[Alternate interior Angles]**

(ii) $\angle GED = 126°$

$\Rightarrow$ $\angle GEF + \angle FED = 126°$

$\Rightarrow$ $\angle GEF + 90° = 126°$

[Since, $EF \perp CD$ and $\angle FED = 90°$]

$\Rightarrow$ $\angle GEF = 126° - 90° = 36°$

(iii) $\angle GEC + \angle GEF + \angle FED = 180°$

[CD is a line]

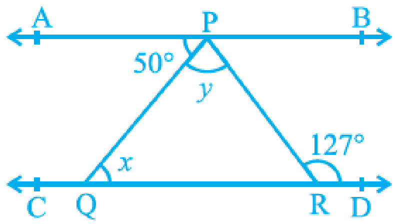
$\Rightarrow$ $\angle GEC + 36° + 90° = 180°$

$\Rightarrow$ $\angle GEC + 126° = 180°$

$\Rightarrow$ $\angle GEC = 180° - 126° = 54°$

Now, $\angle FGE = \angle GEC = 54°$

5. In Figure, if $AB \parallel CD, \angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.



Sol. $x = \angle APQ = 50°$ [Alternate Interior Angles]

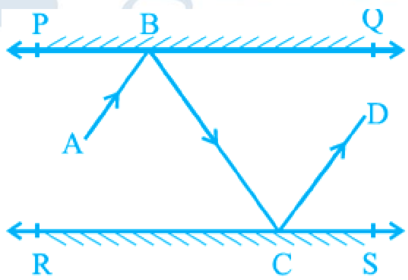
$\angle PRD = x + y = 127°$ [Sum of the two Interior opposite Angles]

$\Rightarrow$ $50° + y = 127°$ [Exterior angle]

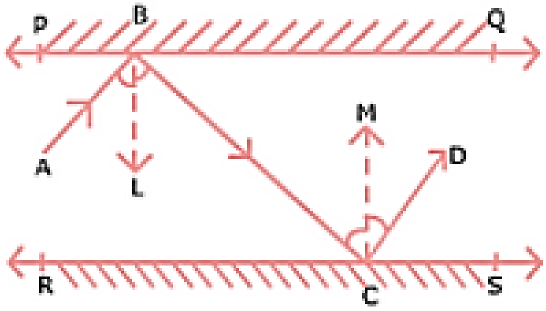
$\Rightarrow$ $y = 127° - 50°$

$\Rightarrow$ $y = 77°$

6. In Figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.



Sol. **Construction :** Draw ray $BL \perp PQ$ and ray $CM \perp RS$.



Proof : $BL \perp PQ, CM \perp RS$ and $PQ \,||\, RS$

$\therefore$ $BL \,||\, CM$

$\angle LBC = \angle MCB$ ……(1)

[Alternate Interior Angles $\angle ABL = \angle LCB$ ….(2)]

(2) [Angle of incidence = Angle of reflection]

$\angle MCB = \angle MCD$ …..(3)

[Angle of incidence = Angle of reflection]

From (1), (2) and (3), we get $\angle ABL = \angle MCD$ ……(4)

Adding (1) and (4), we get

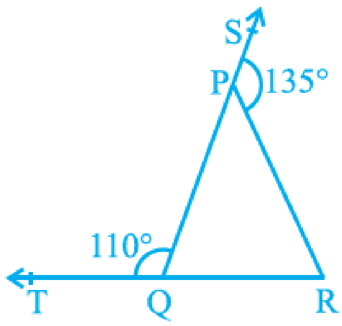
$\angle LBC + \angle ABL = \angle MCB + \angle MCD$

$\Rightarrow$ $\angle ABC = \angle BCD$

But these are alternate interior angles and they are equal.

So, $AB \,||\, CD.$

7. In Figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°,$ find $\angle PRQ.$



Sol. TR is a line $\angle PQT + \angle PQR = 180°$

$\Rightarrow \quad 110° + \angle PQR = 180°$

$\Rightarrow \quad \angle PQR = 180° - 110° = 70°$ …….(1) QS is a line

$\therefore \quad \angle SPR + \angle QPR = 180°$

$\Rightarrow \quad 135° + \angle QPR = 180°$

$\Rightarrow \quad \angle QPR = 180° - 135° = 45°$ ……(2)

In $\Delta PQR, \ \angle PQR + \angle QPR + \angle PRQ = 180°$

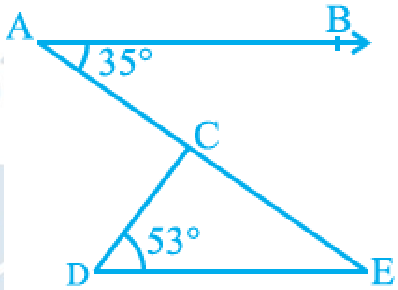
[The sum of all the angles of a triangle is $180°$]

$\Rightarrow \quad 70° + 45° + \angle PRQ = 180°$ [Using (1) and (2)]

$\Rightarrow \quad 115° + \angle PRQ = 180°$

$\Rightarrow \quad \angle PRQ = 180° - 115° = 65°$

8. In Figure, if $AB \parallel DE, \angle BAC = 35°$ and $\angle CDE = 53°,$ find $\angle DCE.$



Sol. $\angle DEC = \angle BAC = 35°$ ……(1)

[Alternate Interior Angles]

$\angle CDE = 53°$ …..(2)

[Given]

In $\Delta CDE, \angle CDE + \angle DEC + \angle DCE = 180°$

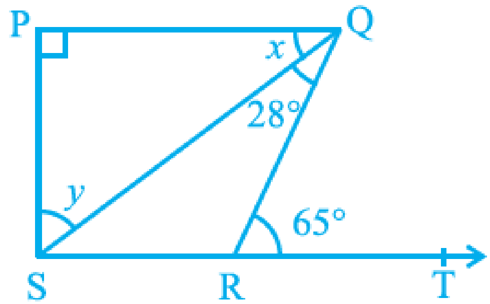
[The sum of all the angles of a triangle is $180°$.]

$\Rightarrow \quad 53° + 35° = \angle CDE + \angle DEC$

$\Rightarrow \quad 88° + \angle DCE = 180°$

$\Rightarrow \quad \angle DCE = 180° - 88° = 92°$

9. In Figure, if $PQ \perp PS, PQ \parallel SR, \angle SQR = 28°$ and $\angle QRT = 65°,$ then find the values of x and y.



Sol. $\angle QRT = \angle RST + \angle QSR$

[The exterior angle is equal to the sum of the two interior opposite angles]

$\Rightarrow$ $65° = 28° + \angle QSR$

$\Rightarrow$ $\angle QSR = 65° - 28° = 37°$

$\Rightarrow$ $PQ \perp SP$

$\Rightarrow$ $\angle QPS = 90°$

$\therefore$ $PQ \parallel SR$

$\therefore$ $\angle QPS + \angle PSR = 180°$

[The sum of consecutive interior angles on the same side of the transversal is $180°$]

$\Rightarrow$ $90° + \angle PSR = 180°$

$\Rightarrow$ $\angle PSR = 180° - 90° = 90°$

$\Rightarrow$ $\angle PSQ + \angle QSR = 90°$

$\Rightarrow$ $y + 37° = 90°$

$\Rightarrow$ $y = 90° - 37° = 53°$

In $\Delta PQS, \angle PQS + \angle QSP + \angle QPS = 180°$

[The sum of all the angles of a triangle is $180°$]

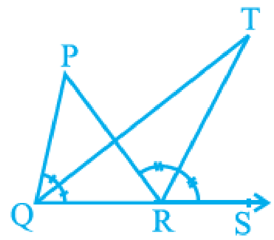
$\Rightarrow$ $x + y + 90° = 180°$

$\Rightarrow$ $x + 53° + 90° = 180°$

$\Rightarrow$ $x + 143° = 180°$

$\Rightarrow$ $x = 180° - 143° = 37°.$

10. In Figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.



Sol. $\angle TRS$ is an exterior angle of ΔTQR

$\angle TRS = \angle TQR + \angle QTR$ ……(1)

[Since, the exterior angle is equal to the sum of the two interior opposite angles.]

$\therefore$ $\angle PRS$ ia an exterior angle of ΔPQR

$\angle PRS = \angle PQR + \angle QPR$ …..(2)

[Since, the exterior angle is equal to the sum of the two interior opposite angles.]

$\Rightarrow$ $2\angle TRS = 2\angle TQR + \angle QPR$

[QT is the bisector of $\angle PQR$ and RT is the bisector of $\angle PRS$]

$\Rightarrow$ $2(\angle TRS - \angle TQR) = \angle QPR$ …..(3)

From (1), $\angle TRS - \angle TQR = \angle QTR$ …..(4)

From (3) and (4), we obtain

$2\angle QTR = \angle QPR$

$\Rightarrow$ $\angle QTR = \frac{1}{2}\angle QPR.$

Hence proved.