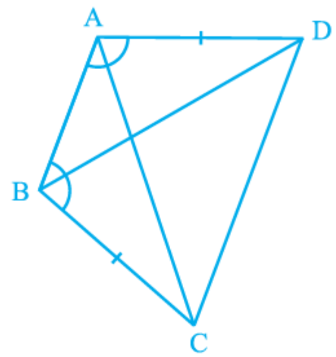
1. ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$ (see figure). Prove that

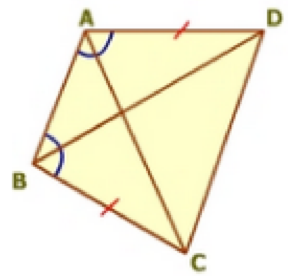


(i) $\Delta ABD \cong \Delta BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$.

Sol. **Given :** ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$.



To prove :

(i) $\Delta ABD \cong \Delta BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$.

Proof : **(i)** **In ΔABD and ΔBAC,**

$AD = BC$ **[Given]**

$AB = BA$ **[AB is given]**

$\angle DAB = \angle CBA$ **[Given]**

$\therefore$ $\Delta ABD \cong \Delta BAC$ **[SAS criterion]**

(ii) In $\Delta ABD \cong \Delta BAC$

$\angle DAB = \angle CBA$

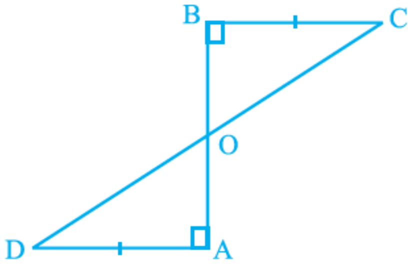
$\therefore$ $BD = AC$ **[CPCT]**

(iii) $\Delta ABD = \Delta BAC$

$AD = BC$ **[Given]**

$\therefore$ $\angle ABD = \angle BAC$ **[CPCT]**

2. AD and BC are equal perpendiculars to a line segment AB (see Figure). Show that CD bisects AB.



Sol. **Given :** AD and BC are equal perpendiculars to a line segment AB.

To Prove : CD bisects AB.

Proof : In ΔOAD and ΔOBC

$AD = BC$ [Given]

$\angle OAD = \angle OBC$ [Each $= 90°$]

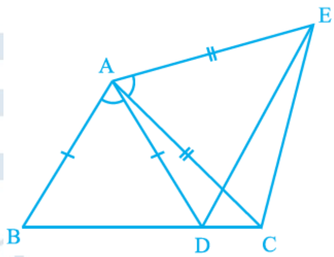
$\angle AOD = \angle BOC$ [Vertically Opposite Angles]

$\therefore$ $\Delta OAD \cong \Delta OBC$ [AAS rule]

$OA = OB$ [CPCT]

$\therefore$ CD bisects AB.

3. In Figure, AC = AE, AB = AD and $\angle BAD = \angle EAC$. Show that BC = DE.



Sol. **Given : In figure, $AC = AE, AB = AD$ and $\angle BAD = \angle EAC$.**

To Prove : $BC = DE$.

Proof : In ΔABC and ΔADE,

$AB = AD$ **[Given]**

$AC = AE$ **[Given]**

$\angle BAD = \angle EAC$ **[Given]**

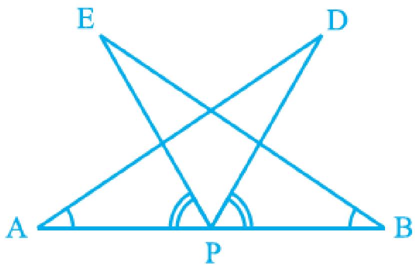
$\Rightarrow$ $\angle BAD + \angle DAC = \angle DAC + \angle EAC$ **[Adding $\angle DAC$ to both sides]**

$\Rightarrow$ $\angle BAC = \angle DAE$

$\therefore$ $\Delta ABC \cong \Delta ADE$ **[SAS Rule]**

$\therefore$ $BC = DE$. **[CPCT]**

4. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ (see Figure). Show that



(i) $\Delta DAP \cong \Delta EBP$

(ii) $AD = BE$

Sol. **Given : AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that** $\angle BAD = \angle ABE$ **and** $\angle EPA = \angle DPB.$

To Prove :

(i) $\Delta DAP \cong \Delta EBP$

(ii) $AD = BE$

Proof: (i) In ΔDAP and $\Delta EBP,$

$AP = BP$ [Since P is the midpoint of the line segment AB]

$\angle DAP = \angle EBP$ [Given]

$\angle EPA = \angle DPB$ [Given]

$\Rightarrow$ $\angle EPA + \angle EPD = \angle EPD + \angle DPB$ [Adding $\angle EPD$ to both sides]

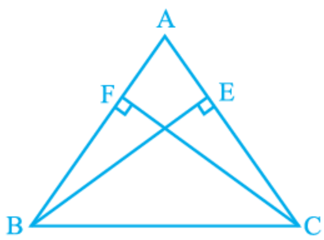
$\Rightarrow$ $\angle APD = \angle BPE$ [ASA Rule]

$\therefore$ $\Delta DAP \cong \Delta EBP$

(ii) $\Delta DAP \cong \Delta EBP$ [From (1) above]

$\therefore$ $AD = BE.$ [CPCT]

5. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see figure). Show that these altitudes are equal.



Sol. **Given :** ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively.

To Prove : BE = CF.

Proof : ABC is an isosceles triangle

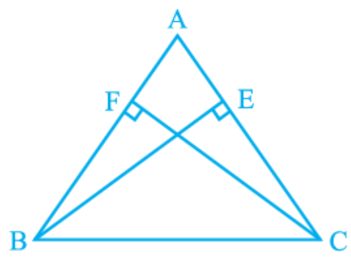
$\therefore$ $AB = AC$

$\therefore$ $\angle ABC = \angle ACB$ ….. (1) [Angles opposite to equal sides of a triangle are equal]

In ΔBEC and ΔCFB,

	$\angle BEC = \angle CFB$	[Each $= 90°$]
	$BC = CB$	[Common]
	$\angle ECB = \angle FBC$	[From (1)]
$\therefore$	$\Delta BEC \cong \Delta CFB$	[By ASA rule]
$\therefore$	$BE = CF$	[CPCT]

6. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see figure). Show that



(i) Δ ABE $\cong$ Δ ACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.

Sol. **Given :** ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal.

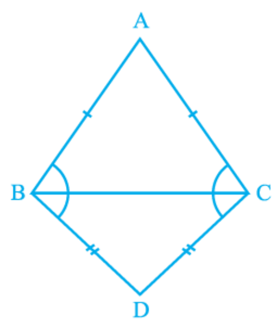
To Prove : (i) Δ ABE $\cong$ Δ ACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.

Proof :

(i)	ΔABE and ΔACF	
	$BE = CF$	[Given]
	$\angle BAE = \angle CAF$	[Common]
	$\angle AEB = \angle AFC$	[Each $= 90°$]
$\therefore$	$\Delta ABE \cong \Delta ACF$	[By AAS Rule]
(ii)	$\Delta ABE \cong \Delta ACF$	[Proved in (i) above]
$\therefore$	$AB = AC$	[CPCT]
$\therefore$	ΔABC is an isosceles triangle.	

7. ABC and DBC are two isosceles triangles on the same base BC (see figure). Show that ∠ABD = ∠ACD.



Sol. **Given** : ABC and DBC are two isosceles triangles on the same base BC.

To Prove : $\angle ABD = \angle ACD$

Proof : ABC is an isosceles triangle on the base BC

$\therefore$ $\angle ABC = \angle ACB$ ……(1)

DBC is an isosceles triangle on the base BC

$\therefore \quad \angle DBC = \angle DCB$ …….(2)

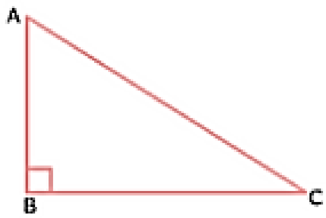
Adding the corresponding sides of (1) and (2), we get

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\Rightarrow \quad \angle ABD = \angle ACD$

8. Show that in a right angled triangle, the hypotenuse is the longest side.

Sol. **Let ABC be a right angled triangle in which** $\angle B = 90°$.



Then, $\angle A + \angle C = 90°$ **[Sum of all the angles of a triangle is** $180°$ **]**

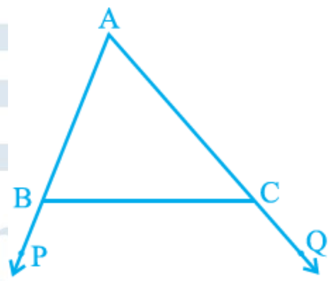
$\therefore \quad \angle B = \angle A + \angle C$

$\therefore \quad \angle B > \angle A$ **and** $\angle B > \angle C$

$\therefore \quad AC > BC$ **[$\therefore$ Side opposite to greater angle is longer and AC > AB]**

$\therefore \quad AC$ **is the longest side, i.e., hypotenuse is the longest side.**

9. In figure, sides AB and AC of Δ ABC are extended to points P and Q respectively. Also, ∠ PBC < ∠ QCB. Show that AC > AB



Sol. **Given :** Sides AB and AC of ΔABC are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB.$

To Prove : $AC > AB.$

Proof : $\angle PBC < \angle QCB$ **[Given]**

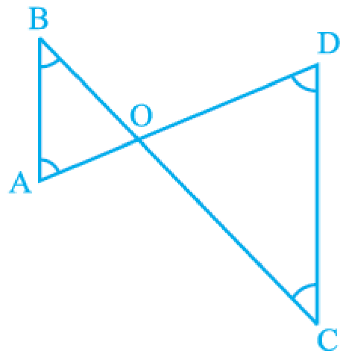
$\Rightarrow \quad -\angle PBC > -\angle QCB$

$\Rightarrow \quad 180° - \angle PBC > 180° - \angle QCB$

$\Rightarrow \quad \angle ABC > \angle ACB$

$\therefore \quad AC > AB$ **[Since the side opposite to the greater angle is longer]**

10. In figure, ∠ B < ∠ A and ∠ C < ∠ D. Show that AD < BC.



Sol. **Given : In figure,** $\angle B < \angle A$ **and** $\angle C < \angle D.$

To Prove : $AD < BC$ **[Given]**

Proof : $\angle B < \angle A$

$\therefore$ $\angle A > \angle B$

$OB > OA$ ……(1) **[Side opposite to greater angle is longer]**

$\angle C < \angle D$ **[Given]**

$\angle D > \angle C$

$OC > OD$ ……(2) **[Side opposite to greater angle is longer]**

From (1) and (2), we get

$OB + OC > OA + OD$

$\Rightarrow$ $BC > AD$

$\Rightarrow$ $AD < BC.$