1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Let ABCD be a quadrilateral in which $\angle A : \angle B : \angle C : \angle D = 3:5:9:13$

Sum of the ratios $= 3+5+9+13 = 30$

Also, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

Sum of all the angles of a quadrilateral is 360°

$\therefore \quad \angle A = \frac{3}{30} \times 360^\circ = 36^\circ$

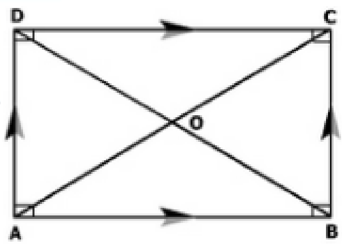
$\angle B = \frac{5}{30} \times 360^\circ = 60^\circ$

$\angle C = \frac{9}{30} \times 360^\circ = 108^\circ$

and $\quad \angle D = \frac{13}{30} \times 360^\circ = 156^\circ$

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol.



Given : In parallelogram ABCD, AC = BD

To Prove : Parallelogram ABCD is a rectangle.

Proof : In ΔACB and ΔBDA,

$AC = BD$ [Given]

$AB = BA$ [Common]

$BC = AD$ [Opposite sides of the parallelogram ABCD]

$\Delta ACB \cong \Delta BDA$ [SSS Rule]

$\therefore \quad \angle ABC = \angle BAD$ …..(1) [CPCT]

Again $AD \,||\, BC$ Opposite sides of parallelogram ABCD

$AD \,||\, BC$ and the transversal AB intersects them.

$\therefore \quad \angle BAD + \angle ABC = 180^\circ$ …..(2)

[Sum of consecutive interior angles on the same side of the transversal is 180°]

From (1) and (2),

$\angle BAD = \angle ABC = 90^\circ$

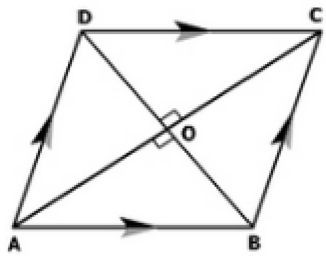
$\therefore \quad \angle A = 90^\circ$ and $\angle C = 90^\circ$

Similarly we can prove $\angle B = 90^\circ$ and $\angle D = 90^\circ$

Parallelogram ABCD is a rectangle.

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Sol. **Given :** ABCD is a quadrilateral where diagonals AC and BD intersect each other at right angles at O.



To Prove : Quadrilateral ABCD is a rhombus.

Proof : In ΔAOB and ΔAOD,

$AO = AO$ [Common]

$OB = OD$ [Given $\angle AOB = \angle AOD$ Each $= 90^\circ$]

$\therefore \quad \Delta AOB = \Delta AOD$ [SAS Rule]

$AB = AD$ …….(1) [CPCT]

Similarly, we can prove that

$AB = BC$ …..(2)

$BC = CD$ …..(3)

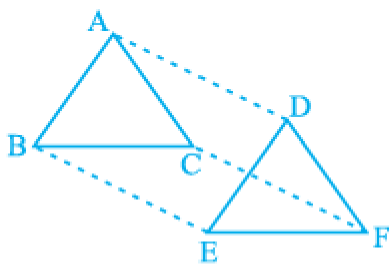
$CD = AD$ …..(4)

From (1), (2), (3) and (4), we obtain

$AB = BC = CD = DA$

$\therefore$ Quadrilateral ABCD is a rhombus.

4. In ΔABC and $\Delta DEF, AB = DE, AB \parallel DE, BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see Figure). Show that



(i) Quadrilateral ABED is a parallelogram

(ii) Quadrilateral BEFC is a parallelogram

(iii) $AD \parallel CF$ and $AD = CF$

(iv) Quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) $\Delta ABC \cong \Delta DEF$.

Sol. **Given : In** ΔABC **and** $\Delta DEF, AB = DE, AB \parallel DE, BC = EF$ and $BC \parallel EF$.

Vertices A, B and C are joined to vertices D, E and F respectively.

To Prove :

(i) Quadrilateral ABED is a parallelogram

(ii) Quadrilateral BEFC is a parallelogram

(iii) $AD \parallel CF$ and $AD = CF$

(iv) Quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) $\Delta ABC \cong \Delta DEF$.

Proof : (i) In quadrilateral ABED,

$AB = DE$ and $AB \parallel DE$ [Given]

$\therefore$ Quadrilateral ABED is a parallelogram

(ii) In quadrilateral BEFC,

$BC = EF$ and $BC \parallel EF$ [Given]

$\therefore$ Quadrilateral BEFC is a parallelogram.

(iii) ABED is a parallelogram [From (i)]

$\therefore$ $AD \parallel BE$ ……(1)

[$\because$ Opposite sides of a parallelogram are parallel and equal.]

BEFC is a parallelogram [From (ii)]

$\therefore$ $BE \parallel CF$ and $BE = CF$ …….(2)

[$\because$ Opposite sides of a parallelogram are parallel and equal.]

From (1) and (2), we obtains $AD \parallel CF$ and $AD = CF$

(iv) In quadrilateral ACFD,

$AD \parallel CF$ and $AD = CF$ [Proved in (iii)]

$\therefore$ Quadrilateral ACFD is a parallelogram

(v) ACFD is a parallelogram [Proved in (iv)]

$AC \parallel DF$ and $AC = DF$

[$\because$ In a parallelogram opposite sides are parallel and of equal length]

(vi) In ΔABC and ΔDEF

$AB = DE$ [$\because$ ABED is a parallelogram]

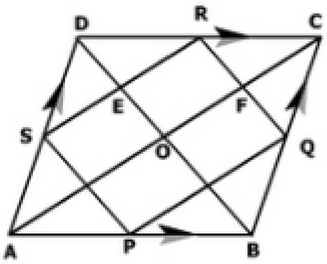
$BC = EF$ [$\because$ BEFC is a parallelogram]

$AC = DF$ [Proved in (v)]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [SSS Rule]

5. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol.



Given : ABCD is a rhombus. P, Q, R, S are the mid-points of AB, BC, CD, DA respectively. PQ, QR, RS and SP are joined.

To Prove : PQRS is a rectangle.

Construction : Join AC and BD.

Proof : In triangles RDS and PBQ.

$DS = QB$ [$\because$ Halves of opposite sides of parallelogram ABCD which are equal]

$DR = PB$ [$\because$ Halves of opposite sides of parallelogram ABCD which are equal]

$\angle SDR = \angle QBP$ [$\because$ Opposite angles of parallelogram ABCD which are equal]

$\therefore$ $\Delta RDS \cong \Delta PBQ$ [$\because$ SAS Axiom]

$\therefore$ $SR = PQ$ [$\because$ c.p.c.t]

In triangles RCQ and PAS,

$RC = AP$ [$\because$ Halves of opposite sides of parallelogram ABCD which are equal]

$CQ = AS$ [$\because$ Halves of opposite sides of parallelogram ABCD which are equal]

$\angle RCQ = \angle PAS$ [$\because$ Opposite angles of parallelogram ABCD which are equal]

$\therefore$ $\Delta RCQ \cong \Delta PAS$

$RQ = SP$ [CPCT]

$\therefore$ In $PQRS$,

$SR = PQ$ and $RQ = SP$

$\therefore$ $PQRS$ is a parallelogram,

In ΔCDB,

$\therefore$ R and Q are the mid-points of DC and CB respectively.

$RQ \parallel DB \Rightarrow RF \parallel EO.$

Similarly, $RE \parallel FO$

$\therefore$ OFRE is a parallelogram

$\therefore$ $\angle R = \angle EOF = 90°$

[$\because$ Opposite angles of a parallelogram are equal and diagonals of a rhombus intersect at $90°$]

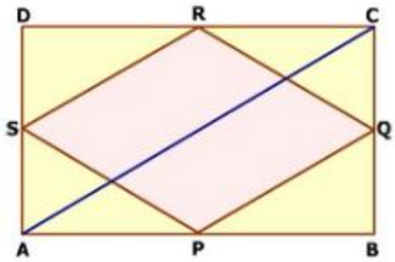
Thus PQRS is a rectangle.

6. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Sol. **Given :** ABCD is a rectangle. P, Q, R and S are mid-points of AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To Prove : Quadrilateral PQRS is a rhombus.

Construction : Join AC.



Proof : In ΔABC,

$\therefore$ P and Q are the mid-points of AB and BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ …..(1)

In ΔADC,

$\therefore$ S and R are the mid-points of AD and DC respectively.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ ……(2)

From (1) and (2),

$PQ \parallel SR$ and $PQ = SR$

$\therefore$ Quadrilateral PQRS is a parallelogram …..(3)

In rectangle ABCD,

AD = BC [Opposite sides $\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$]

$\therefore$ Halves of equals are equal

$\Rightarrow$ $AS = BQ$

In ΔAPS and ΔBPQ,

$AP = BP$ [$\because$ P is the mid-point of AB]

$AS = BQ$ [Proved above]

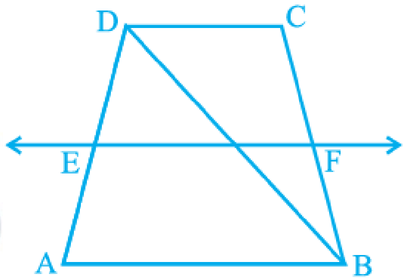
$\angle PAS = \angle PBQ$ [Each $= 90°$]

$\therefore$ $\Delta APS \cong \Delta BPQ$ [SAS Axiom]

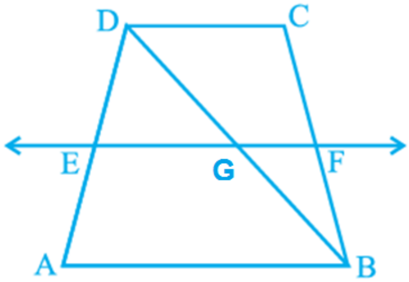
$\therefore$ $PS = PQ$ …..(4) [c.p.c.t.]

In view of (3) and (4), PQRS is a rhombus.

7. $ABCD$ is a trapezium in which $AB \parallel DC, BD$ is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Figure). Show that F is the mid-point of BC.



Sol. **Given:** ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at R.



To Prove : F is the mid-point of BC.

Proof : Let DB intersect EF at G.

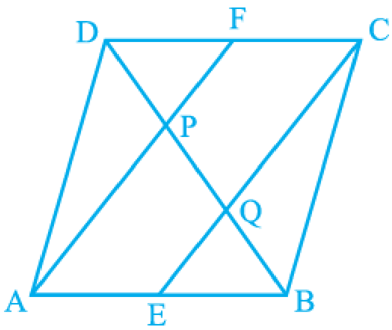
In $\Delta DAB, E$ is the mid-point of DA and $EG \parallel AB$

$\therefore$ G is the mid-point of DB. [By converse of mid-point theorem]

Again, in $\Delta BDC, G$ is the mid-point of BD and $GF \parallel AB \parallel DC$

$\therefore$ F is the mid-point of BC. [By converse of mid-point theorem]

8. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Figure). Show that the line segments AF and EC trisect the diagonal BD**.**



Sol. **Given :** In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively.

To Prove : Line segment AF and EC trisect the diagonal BD.

Proof : $AB \parallel DC$ ($\because$ Opposite sides of parallelogram ABCD)

$\therefore \quad AE \parallel FC$ …….(1)

$\therefore \quad AB = DC$ ($\because$ Opposite sides of parallelogram ABCD)

$\frac{1}{2}AB = \frac{1}{2}DC$ (Halves of equals are equal)

$\Rightarrow \quad AE = CF$ ……(2)

In view of (1) and (2),

$AECF$ is a parallelogram

$\therefore \quad EC \parallel AF$ …….(3)

(Opposite sides of parallelogram AECF)

In $\Delta DBC, F$ is the mid-point of DC

and $FP \parallel CQ$ ($\because$ $EC \parallel AF$)

$\therefore$ P is the mid-point of DQ (By converse of mid-point theorem)

$\Rightarrow \quad DP = PQ$ …….(4)

Similarly, in ΔBAP,

$BQ = PQ$ …….(5)

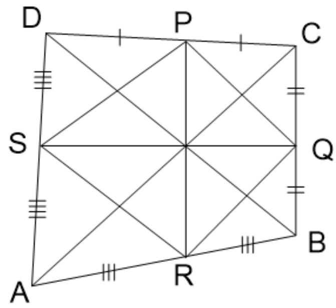
From (4) and (5), we obtain

$DP = PQ = BQ$

$\Rightarrow$ Line segments AF and EC trisect the diagonal BD.

9. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Sol. **Given :** ABCD is a quadrilateral. P, Q, R and S are the mid-points of the sides DC, CB, BA and AD respectively.



To Prove : PR and QS bisect each other.

Construction : Join PQ, QR, RS, SP, AC and BD.

Proof : In ΔABC,

$\therefore$ R and Q are the mid-points of AB and BC respectively.

$\therefore$ $RQ \,||\, AC$ and $RQ = \frac{1}{2}AC$.

Similarly, we can show that

$PS \,||\, AC$ and $PS = \frac{1}{2}AC$.

$\therefore$ $RQ \,||\, PS$ and $RQ = PS$.

Thus, pair of opposite sides of a quadrilateral PQRS are parallel and equal.

$\therefore$ $PQRS$ is a parallelogram.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ PR and QS bisect each other.

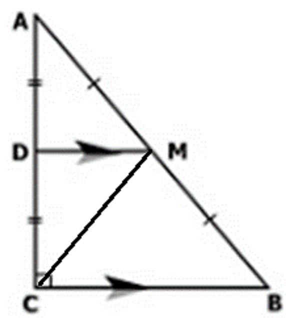
10. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2}AB$

Sol.



Given : ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D.

To Prove :

(i) D is the mid-point of AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2}AB$

Proof : **(i)** In $\Delta ACB, M$ is the mid-point of AB and $MD \parallel BC$

$\therefore$ D is the mid-point of AC [By converse of mid-point theorem]

(ii) $MD \parallel BC$ and AC intersects them

$\therefore \angle ADM = \angle ACB$ [Corresponding angles]

But $\angle ACB = 90^\circ$ [Given]

$\therefore \angle ADM = 90^\circ$

$\Rightarrow MD \perp AC$

(iii) Now, $\angle ADM + \angle CDM = 180^\circ$ [Linear Pair Axiom]

$\therefore \angle ADM = \angle CDM = 90^\circ$

In ΔADM and ΔCDM,

$AD = CD$ [$\therefore$ D is the mid-point of AC]

$\angle ADM = \angle CDM$ [Each $= 90^\circ$]

$DM = DM$ [Common]

$\therefore \Delta ADM \cong \Delta CDM$ [SAS Rule]

$\therefore MA = MC$ [CPCT]

But M is the mid-point of AB

$\therefore MA = MB = \frac{1}{2}AB$

$\therefore MA = MC = \frac{1}{2}AB$

$\Rightarrow CM = MA = \frac{1}{2}AB$