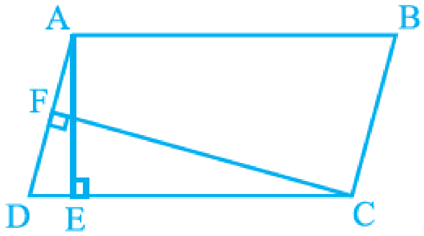
1. In Figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Sol.

Area of parallelogram $ABCD = CD \times AE = 16 \times 8 cm^2$

(∵ AB = CD, ABCD is a parallelogram)

$= 128 cm^2$ …..(1)

Also area of parallelogram $ABCD = AD \times CF = AD \times 10 cm^2$ ……(2)

From (1) and (2), we get

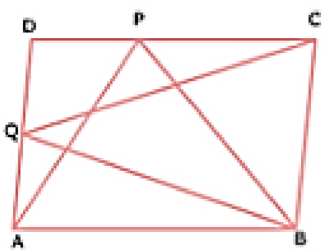
$AD \times 10 = 128$

$\Rightarrow \quad AD = \frac{128}{10}$

$\Rightarrow \quad AD = 12.8 cm$

2. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

Sol.



Given: P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD.

To Prove: Area $(\Delta APB) =$ Area (ΔBQC).

Proof : ΔAPB and parallelogram ABCD are on the same base AB and between the same parallels AB and DC.

$\therefore$ Area $(\Delta APB) = \frac{1}{2}$ Area of parallelogram ABCD …….(1)

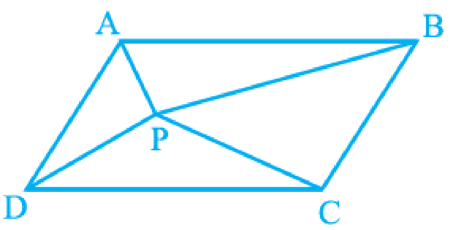
$\therefore$ ΔBQC and parallelogram ABCD are on the same base BC and between the same parallels BC and AD.

$\therefore$ $Area(\Delta BQC) = \frac{1}{2}$ Area (parallelogram ABCD) ……(2)

From (1) and (2),

$Area(\Delta APB) = Area(\Delta BQC)$.

3. In Figure, P is a point in the interior of a parallelogram ABCD. Show that



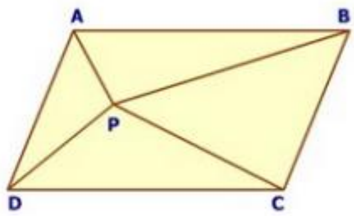
(i) $ar(APB) + ar(PCD) = \frac{1}{2} ar(ABCD)$

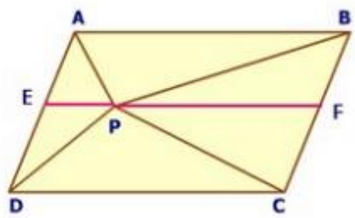
(ii) $ar(APD) + ar(PBC) = ar(APB) + ar(PCD)$

[Hint : Through P, draw a line parallel to AB.]

Sol. **Given: P is a point in the interior of a parallelogram ABCD.**

To Prove:





(i) $ar(APB) + ar(PCD) = \frac{1}{2} ar(ABCD)$

(ii) $ar(APD) + ar(PBC) = ar(APB) + ar(PCD)$

Construction: Through P, draw a line EF parallel to AB.

Proof: **(i)** $EF \parallel AB$ …..(1) (by construction)

$AD \parallel BC$ ($\because$ Opposite sides of a parallelogram are parallel)

$\therefore$ $AE \parallel BF$ …..(2)

In view of (1) and (2),

Quadrilateral ABFE is a parallelogram (A quadrilateral is a parallelogram if it's opposite sides are parallel)

Similarly, quadrilateral CDEF is a parallelogram.

$\therefore$ ΔAPB and parallelogram ABFE are on the same base AB and between the same parallels AB and EF.

$\therefore$ Area $(\Delta APB) = \frac{1}{2}$ Area (parallelogram ABFE) ……(3)

$\therefore$ ΔPCD and parallelogram CDEF are on the same base DC and between the same parallels DC and EF.

$\therefore$ Area $(\Delta PCD) = \frac{1}{2}$ Area (parallelogram CDEF) …..(4)

Adding (3) and (4), we get

Area (ΔAPB) + Area (ΔPCD) = $\frac{1}{2}$ Area (parallelogram ABFE) $+\frac{1}{2}$

Area (parallelogram CDEF) $= \frac{1}{2}$[Area (parallelogram ABFE) + Area (parallelogram CDEF)]

$= \frac{1}{2}$ Area (parallelogram ABCD)

Area (ΔAPB) + Area (ΔPCD) = $\frac{1}{2}$ Area (parallelogram ABCD) …… (5)

(ii) Area (ΔAPD) + Area (ΔPBC) = Area (parallelogram ABCD) − Area ΔAPB + Area ΔPCD)]

$= 2\left[Area(\Delta APB) + Area(\Delta PCD) \right] - Area\,\Delta APB + Area\,\Delta PCD)]$ (From (5))

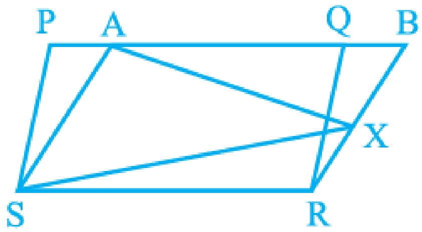
= Area (ΔAPB) + Area (ΔPCD)

$\therefore$ Area (ΔAPD) + Area (ΔPBC) = Area (ΔAPB) + Area (ΔPCD)

4. In Figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i) ar (PQRS) = ar (ABRS)

(ii) $ar(AXS) = \frac{1}{2} ar(PQRS)$



Sol. **Given:** PQRS and ABRS are parallelograms and X is any point on side BR.

To Prove:

(i) ar (PQRS) = ar (ABRS)

(ii) $ar(AXS) = \frac{1}{2} ar(PQRS)$

Proof:

(i) ar(ABRS) = ar(PQRS) ……..(1)

[$\because$ Parallelograms on the same base and between the same parallels are equal in area]

(ii) ΔAXS and parallelogram ABRS are on the same base AS and between the same parallels AS and BR.

$\therefore$ $\text{ar}(\Delta AXS) = \frac{1}{2}$ ar(parallelogram ABRS)

$= \frac{1}{2}$ ar(parallelogram PQRS). [From (i)]

5. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Sol. The field is divided into three triangles. The three triangles are:

(i) ΔAPS **(ii)** ΔAPQ **(iii)** ΔAQR

$ar(\Delta PSA) + ar(\Delta PAQ) + ar(\Delta QRA) = ar(PQRS)$ …..(i)

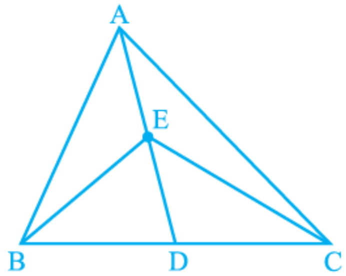
If a parallelogram & a triangle are on the same base and between the same parallel then area of triangle is half area of parallelogram.

$\therefore$ $ar(\Delta PAQ) = \frac{1}{2} ar(PQRS)$ ……(2)

So from (1) and (2)

$ar(\Delta PSA) + ar(\Delta QRA) = ar(\Delta PAQ) = \frac{1}{2} ar(PQRS)$

6. In Figure, E is any point on median AD of a Δ ABC. Show that ar (ABE) = ar (ACE).



Sol. **Given:** E is any point on median AD of a ΔABC.

To Prove: area (ΔABC) = area (ΔACE).

Proof: In $\Delta ABC,$ AD is a median.

$\therefore$ area (ΔABD) = area (ΔACD) …..(1)

[Median of a triangle divides it into two triangles of equal area]

In $\Delta EBC,$

ED is a median.

$\therefore$ area (ΔEBD) = area (ΔECD) ……(2)

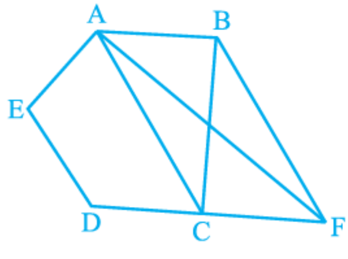
Subtracting (2) from (1), we get

Area (ΔABD) − area (ΔEBD) = area (ΔACD) − area (ΔECD)

$\Rightarrow$ area (ΔABD) − area (ΔEBD) = area (ΔACD) − area (ΔECD)

$\Rightarrow$ area (ΔABE) = area (ΔACE).

7. In Figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that



(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Sol. **Given : ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F.**

To Prove :

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Proof : (i) ΔACB and ΔACF are on the same base AC and between the same parallels AC and BF.

[$\because$ AC || BF (given)]

$\therefore$ ar (ΔACB) = ar (ΔACF)

[$\because$ Two triangles on the same base (or equal bases) and between the same parallels are equal in area]

$\because$ ar (ΔACB) = ar (ΔACF)

$\Rightarrow$ ar (ΔACB) + ar (AEDC) = ar (ΔACF) + ar (AEDC) [Add area (AEDC) on both sides]

$\Rightarrow$ ar (ABCDE) = ar (AEDF)

$\Rightarrow$ ar (AEDF) = ar (ABCDE).

8. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

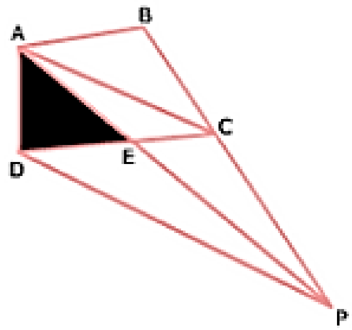
Sol. Let ABCD be the plot of land in the shape of a quadrilateral. Let the portion ADE be taken over by the Gram Panchayat of the village from one corner D to construct a Health Centre.

Join AC.

Draw a line through D parallel to AC to meet BC produced in P.

Then Itwaari must be given the land ECP adjoining his plot so as to forma triangular plot ABP as then

$ar(\Delta ADE) = ar(\Delta PEC)$



Proof : ΔDAP and ΔDCP are on the same base DP and between the same parallels DP and AC.

$\therefore$ **ar (ΔDAP) = ar (ΔDCP)**

[Two triangles on the same base (or equal bases) and between the same parallels are equal in area)]

$\Rightarrow$ **ar (ΔDAP) − ar (ΔDEP) = ar (ΔDCP) − ar (ΔDEP)**

[Subtract ar (ΔDEP) from both sides]

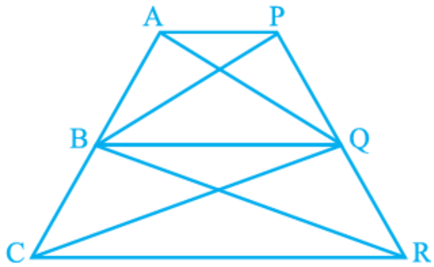
Ar (ΔADE) = ar (ΔPCE)

$\Rightarrow$ **ar (ΔADE) + ar $(ABCE)$ = ar (ΔPCE) + ar $(ABCE)$**

[Add ar (ABCE) both sides]

$\Rightarrow$ **ar (ABCD) = ar (ΔABP)**

9. In Figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).



Sol. **Given :** AP || BQ || CR.

To Prove : ar (ΔAQC) = ar (ΔPBR).

Proof : ΔBAQ and ΔBPQ are on the same base BQ and between the same parallels BQ and AP.

$\therefore$ **ar** $(\Delta BAQ) = ar\ (\Delta BPQ)$ **…….(1)**

[Two triangles on the same base (or equal bases) and between the same parallels are equal in area]

ΔBCQ and ΔBQR are on the same base BQ and between the same parallels BQ and CR.

Ar $(\Delta BCQ) =$ ar (ΔBQR) …….(2)

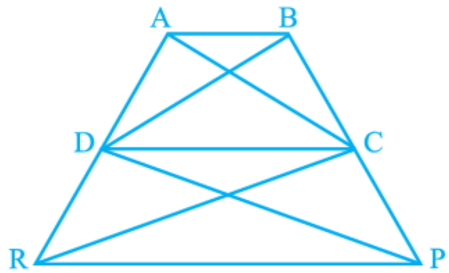
[Two triangles on the same base (or equal bases) and between the same parallels are equal in area]

Adding the corresponding sides of (1) and (2), we get

ar (ΔBAQ) + ar (ΔBCQ) = ar (ΔBPQ) + ar (ΔBQR)

$\Rightarrow$ ar $(\Delta ACQ) =$ ar (ΔPBR)

10. In Figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Sol. **Given:** ar (ΔDRC) = ar (ΔDPC) and ar (ΔBDP) = ar (ΔARC).

To Prove: Both the quadrilateral ABCD and DCPR are trapeziums.

Proof: ar (ΔDRC) = ar (ΔDPC) (given) …….(1)

But ΔDRC and ΔDPC are on the same base DC.

$\therefore$ ΔDRC and (ΔDPC) will have equal corresponding altitudes.

and ΔDRC and (ΔDPC) will lie between the same parallels.

$\therefore$ $DC \| RP$ [$\because$ A quadrilateral is a trapezium if exactly one pair of opposite sides is parallel.]

$\Rightarrow$ DCPR is a trapezium.

Again, ar (ΔBDP) = ar (ΔARC)

$\Rightarrow$ ar (ΔBDC) + ar (ΔDPC) = ar (ΔADC) + ar (ΔDRC)

$\Rightarrow$ ar (ΔBDC) = ar (ΔADC) (Using (1)

But Δ BDC and Δ ADC are on the same base DC.

$\therefore$ ΔBDC and ΔADC will have equal corresponding altitudes.

And ΔBDC and ΔADC will lie between the same parallels.

$\therefore$ $AB \| DC$

$\Rightarrow$ ABCD is a trapezium.

[$\because$ A quadrilateral is a trapezium if exactly one pair of opposite sides is parallel.]