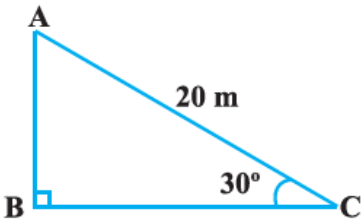
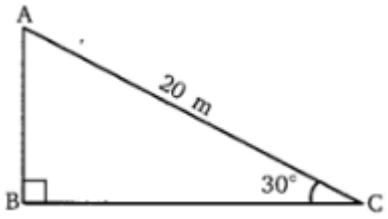
1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is $30°$ (see Figure).



Sol. **Given :** Length of the rope (AC) = 20 m, and $\angle ACB = 30°$

Let height AB of pole be h m.



Then in right ΔABC,

$$\sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{h}{20} \qquad \left[\because \sin 30° \frac{1}{2}\right]$$

$$\Rightarrow \quad h = \frac{20}{2} = 10\,m$$

Hence, height of the pole = 10 m.

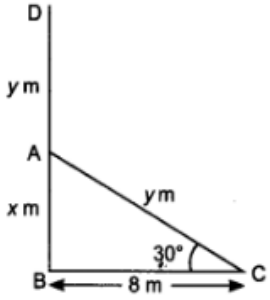
2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle $30°$ with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol. Let DB is a tree and AD is the broken part of it which touches the ground at C.

Given : $\angle ACB = 30°$ and BC = 8 m

Let $AB = xm$ and $AD = ym$

$\therefore$ Now, length of the tree $= (x + y)m$



In ΔABC,

$$\frac{AB}{BC} = \tan 30° \Rightarrow \frac{x}{8} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{8}{\sqrt{3}} \quad \text{.....(i)}$$

and $\frac{AB}{AC} = \sin 30° \Rightarrow \frac{x}{y} = \frac{1}{2}$

$\Rightarrow \quad y = 2x$

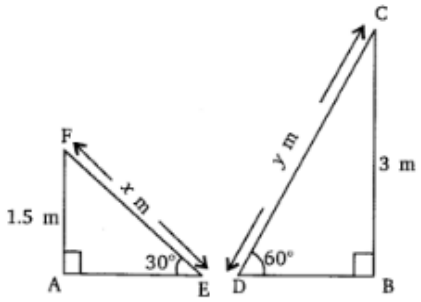
$\Rightarrow \quad y = 2 \times \frac{8}{\sqrt{3}} = \frac{16}{\sqrt{3}}$ [From equation (i)]

Hence, total height of the tree

$$x + y = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}\, m$$

3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of $30°$ to the ground, whereas for elder children, she wants to have a steep slide at a height of 3m, and inclined at an angle of $60°$ to the ground. What should be the length of the slide in each case?

Sol. Let the length of slide for children below the age of 5 years be x m and length of the slide for elder children be y m.



Given :

$AF = 1.5m, BC = 3m, \angle FEA = 30°$ and $\angle CDB = 60°$

In right ΔFAE,

$$\sin 30° = \frac{AF}{EF} = \frac{1.5}{x}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1.5}{x} \quad \Rightarrow \quad x = 3m$$

In right ΔCBD,

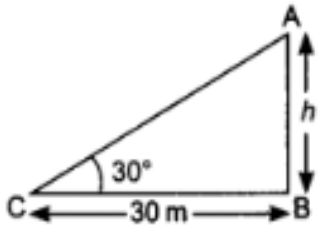
$$\sin 60° = \frac{BC}{CD} = \frac{3}{y}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3}{y} \quad \Rightarrow \quad y = \frac{3 \times 2}{\sqrt{3}} = 2\sqrt{3}\, m.$$

Hence, the length of slide for children below the age of 5 years is 3 m and the length of slide for elder children is $2\sqrt{3}\, m$.

4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is $30°$. Find the height of the tower.

Sol. Let h be the height of the tower

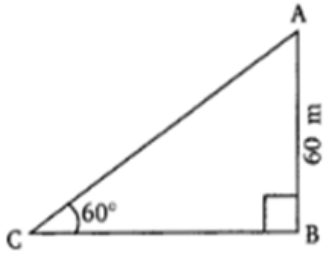


In ΔABC, $\frac{AB}{BC} = \tan 30°$ $\Rightarrow$ $\frac{h}{30} = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\, m$$

5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is $60°$. Find the length of the string, assuming that there is no slack in the string.

Sol. Given : $AB = 60\,m$ and $\angle ACB = 60°$



Let AC be the length of the string.

Then in right ΔABC,

$$\sin 60° = \frac{AB}{AC}$$

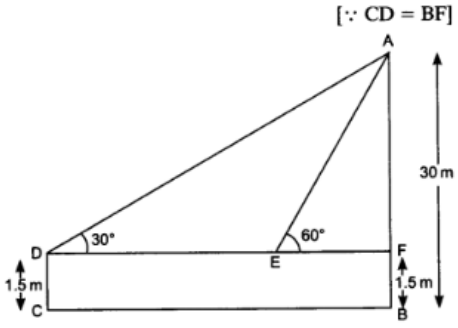
$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

$$\Rightarrow \quad AC = \frac{60 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120 \times \sqrt{3}}{3} = 40\sqrt{3}\,m.$$

Hence, the length of the string is $40\sqrt{3}\,m.$

6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from $30°$ to $60°$ as he walks towards the building. Find the distance he walked towards the building.

Sol. Let AB = height of the building



Given : $\angle ADF = 30°, \angle AEF = 60$

$AF = AB - FB$

$= 30m - 1.5m = 28.5m$

In ΔAFE,

$$\frac{AF}{EF} = \tan 60°$$

$$\Rightarrow \quad \frac{28.5}{EF} = \sqrt{3}$$

$$\Rightarrow \quad EF = \frac{28.5}{\sqrt{3}} m$$

In ΔAFD,

$$\frac{AF}{DF} = \tan 30°$$

$$\Rightarrow \quad \frac{28.5}{DF} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad DF = 28.5\sqrt{3}\, m$$

The distance walked by the boy towards building

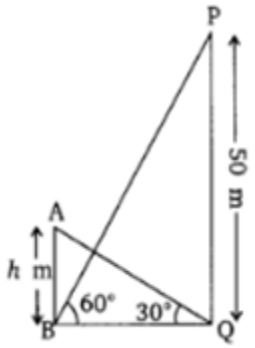
$DE = DF - EF$

$$= 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}} = \frac{28.5 \times 3 - 28.5}{\sqrt{3}} = \frac{28.5(3-1)}{\sqrt{3}}$$

$$= \frac{28.5 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{57\sqrt{3}}{3} = 19\sqrt{3}\, m$$

7. The angle of elevation of the top of a building from the foot of the tower is $30°$ and the angle of elevation of the top of the tower from the foot of the building is $60°$. If the tower is 50 m high, find the height of the building.

Sol. Given : height of the tower AB = 50 m,



Let h m be the height of the building

Then in right ΔABQ,

$$\tan 30° = \frac{AB}{BQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BQ}$$

$$\Rightarrow BQ = h\sqrt{3} \qquad \text{.....(i)}$$

In right ΔPQB,

$$\tan 60° = \frac{PQ}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{50}{BQ}$$

$$\Rightarrow BQ\sqrt{3} = 50$$

$$\Rightarrow h\sqrt{3} \times \sqrt{3} = 50 \qquad \text{[From (i)]}$$

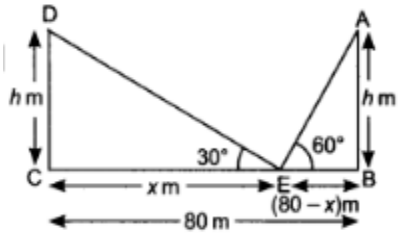
$$\Rightarrow 3h = 50$$

$$\Rightarrow h = \frac{50}{3} = 16\frac{2}{3} m.$$

Hence, the height of the building is $16\frac{2}{3} m.$

8. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are $60°$ and $30°$, respectively. Find the height of the poles and the distances of the point from the poles.

Sol. Let AB = CD = h m [Height of the poles]
Given : BC = 80 m [Width of the road]



Let $CE = x\,m$

$\therefore \quad BE = (80 - x)\,m$

In ΔCDE, $\dfrac{CD}{CE} = \dfrac{h}{x} = \tan 30°$

$\dfrac{h}{x} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad x = \sqrt{3}\,h$ ….(i)

In ΔABE, $\dfrac{AB}{BE} = \tan 60° \Rightarrow \dfrac{h}{80 - x} = \sqrt{3}$

$\Rightarrow \quad h = 80\sqrt{3} - \sqrt{3}x$

$\Rightarrow \quad \sqrt{3}x = 80\sqrt{3} - h$

$\Rightarrow \quad x = \dfrac{80\sqrt{3} - h}{\sqrt{3}}$ ….(ii)

From equation (i) and (ii), we get

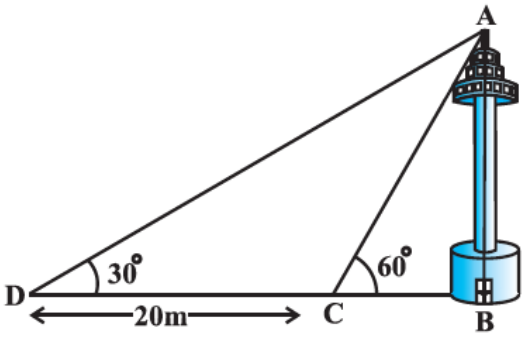
$$\sqrt{3}h = \frac{80\sqrt{3} - h}{\sqrt{3}} \Rightarrow 3h = 80\sqrt{3} - h \Rightarrow 4h = 80\sqrt{3} \; h = 20\sqrt{3}$$

Substituting h in equation (i),

$$x = h\sqrt{3} = 20\sqrt{3} \times \sqrt{3} = 60\,m$$

Hence, position of the point is at a distance of 60 m from pole CD and 20 m from pole AB.

9. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is $60°$. From another point 20 m away from this point on the line join this point to the foot of the tower, the angle of elevation of the top of the tower is $30°$ (see Figure). Find the height of the tower and the width of the canal.



Sol. Let the height of the tower AB be h m and x m be the width of the canal BC.

Then in right ΔABD,

$$\tan 30° = \frac{AB}{BD} = \frac{AB}{DC + CB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20 + h}$$

$$\Rightarrow \quad 20 + x = h\sqrt{3}$$

$$\Rightarrow \quad x = h\sqrt{3} - 20 \qquad \text{.....(i)}$$

In right ΔABC, $\tan 60° = \frac{AB}{BC}$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \qquad \Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \text{.....(ii)}$$

From equations (i) and (ii), we get:

$$h\sqrt{3} - 20 = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \quad h\sqrt{3} - \frac{h}{\sqrt{3}} = 20 \qquad \Rightarrow \quad h\left(\frac{3-1}{\sqrt{3}}\right) = 20$$

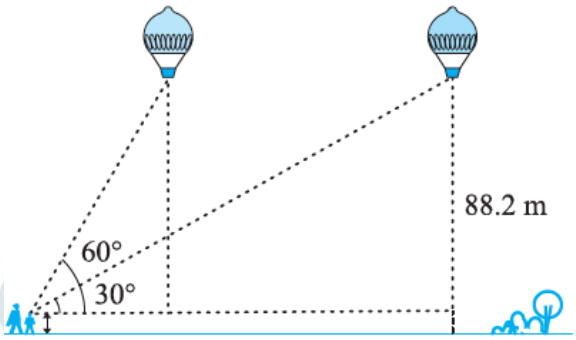
$$\Rightarrow \quad h = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$$

Putting the value of $h = 10\sqrt{3}$ in equation (ii), we get:

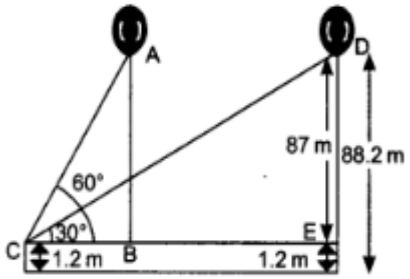
$$x = \frac{h}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10\,m.$$

Hence, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

10. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is $60°$. After some time, the angle of elevation reduces to $30°$ (see Figure). Find the distance travelled by the balloon during the interval.



Sol. Let the first position of the balloon is A and after sometime it will reach to the point D.



The vertical height $ED = AB = (88.2 - 1.2)\,m = 87m.$

In ΔABC, $\quad \frac{AB}{BC} = \tan 60°$

$\Rightarrow \quad \frac{87}{BC} = \sqrt{3}$

$\Rightarrow \quad BC = \frac{87}{\sqrt{3}} = 29\sqrt{3}$

In ΔDEC, $\frac{DE}{CE} = \tan 30°$

$\Rightarrow \quad \frac{87}{CE} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad CE = 87\sqrt{3}\, m$

Distance travelled by the balloon from A to D is BE.

So, $BE = CE - CB$

$= 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}\, m$