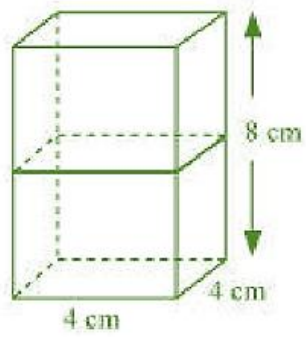
1. 2 cubes each of volume $64\ cm^3$ are joined end to end. Find the surface area of the resulting cuboid.

Sol. Given that: Volume of cube $= 64 cm^3$

$\Rightarrow (Side)^3 = 64 cm^3 \Rightarrow Side = 4 cm$

The sides of cuboids formed by joining the cubes are 4 cm, 4 cm and 8 cm.

Surface area of resulting cuboid



$= 2(lb + bh + hl)$

$= 2(4 \times 4 + 4 \times 8 + 4 \times 8) cm^2$

$= 2(16 + 32 + 32) cm^2$

$= 2(80) cm^2$

$= 160 cm^2$

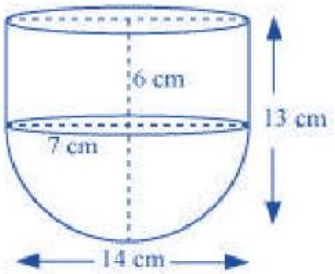
Hence, the surface area of the resulting cuboid $160 cm^2$.

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Sol. Radius of cylinder = 7 cm

Height of cylinder = 13 – 7 = 6 cm

Radius of hemi-sphere = 7 cm



Inner surface area of the vessel = CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 6 + 2 \times \frac{22}{7} \times 7^2$

$= 44(6+7)$

$= 44 \times 13$

$= 572 \text{ cm}^2$

Hence, the inner surface area of the vessel is 572 cm^2.

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

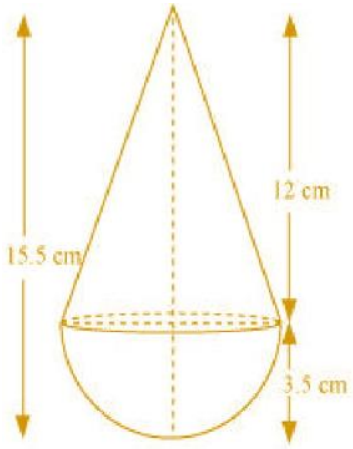
Sol. Radius of cone = 3.5 cm

Height of cone = 15.5 – 3.5 = 12 cm

Radius of hemisphere = 3.5 cm

Slant height of cone (l)

$= \sqrt{r^2 + h^2} = \sqrt{(3.5)^2 + (12)^2} = \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5$



The total surface area of the toy

= CSA of cone + CSA of hemisphere

$= \pi rl + 2\pi r^2$

$= \frac{22}{7} \times 3.5 \times 12.5 + 2 \times \frac{22}{7} \times 3.5^2$

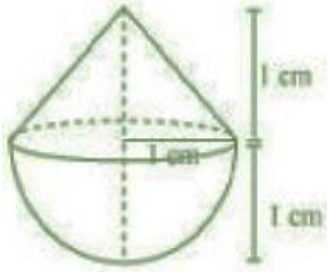
= 137.5 + 77

$= 214.5 \text{cm}^2$

Hence, the total surface area of the toy is 572 cm^2.

4. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Sol. Height of conical part (h) = radius of conical part (r) = 1 cm
Radius of conical part (r) = radius of hemispherical part (r) = 1 cm
Volume of Solid = Volume of conical part + Volume of hemispherical part

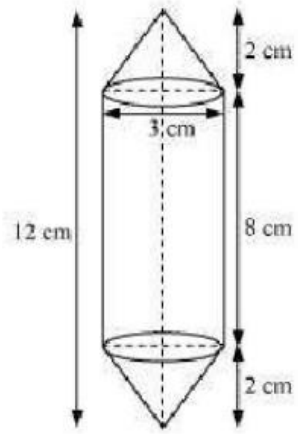


$$=\frac{1}{3}\pi r^2 h+\frac{2}{3}\pi r^3$$

$$=\frac{1}{3}\pi.1^2.1+\frac{2}{3}\pi.1^3=\pi\, cm^3$$

5. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Sol. Height of conical part $(h_1)=2cm$
Radius of conical part (r) = Radius of cylindrical part (r) = 3/2 cm
Height of cylindrical part $(h_2)=12-2\times$ Height of conical part $=12-2\times2=8$ cm
The volume of air contained in the model
= Volume of cylindrical part $+2\times$ Volume of conical part



$$= \pi r^2 h_2 + 2 \times \frac{1}{3} \pi r^2 h_1$$

$$= \pi \left(\frac{3}{2}\right)^2 \times 8 + 2 \times \frac{1}{3} \pi \left(\frac{3}{2}\right)^2 \times 2$$

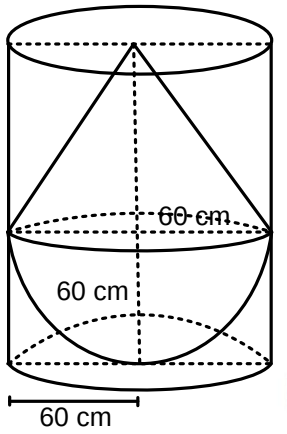
$$= 18\pi + 3\pi = 21\pi = 21 \times \frac{22}{7} = 66\, cm^3$$

6. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Sol. Radius of hemispherical part = Radius of conical part = Radius of cylindrical part = 60 cm

Height of cylindrical part $(h_1) = 180\, cm$, Height of conical part $(h_2) = 120\, cm$

Radius of cylinder $(r) = 60\, cm$



Volume of water left in the cylinder =Volume of cylinder – (Volume of hemisphere + Volume of cone)

$$= \pi r^2 h_1 - \left(\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h_2\right) = \pi (60)^2 \times 180 - \left[\frac{2}{3}\pi (60)^3 + \frac{1}{3}\pi (60)^2 \times 120\right]$$

$$= \pi (60)^2 [180 - (40 + 40)] = \frac{22}{7} \times 60 \times 60 \times 100$$

$$= 1131428.57 \text{ cm}^3 = 1.131 \text{ m}^3$$

Therefore, volume of water left in the cylinder is 1.131 m^3.

7. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol. Radius of metallic sphere $(r_1) = 4.2\ cm$,

Radius of cylinder $(r_2) = 6\ \text{cm}$

Let, the height of cylinder be h

According to question, volume of sphere = volume of cylinder

$$\Rightarrow \frac{4}{3}\pi r_1^3 = \pi r_2^2 h \qquad \Rightarrow \frac{4}{3}\pi (4.2)^3 = \pi (6)^2 h$$

$$\Rightarrow h = \frac{4}{3} \times \frac{4.2 \times 4.2 \times 4.2}{36} = 1.4 \times 1.4 \times 1.4 = 2.74\ cm$$

Hence, the height of cylinder is 2.74 cm.

8. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol. Radius of first sphere $(r_1) = 6\ cm$,

Radius of second sphere $(r_2) = 8\ cm$,

Radius of third sphere $(r_3) = 10\ cm$,

Let the radius of the new sphere = r

According to question, volumes of three spheres = volume of new sphere

$$\Rightarrow \frac{4}{3}\pi (r_1^3 + r_2^3 + r_3^3) = \frac{4}{3}\pi r^3 \qquad \Rightarrow \frac{4}{3}\pi (6^3 + 8^3 + 10^3) = \frac{4}{3}\pi r^3$$

$$\Rightarrow 216 + 512 + 1000 = r^3 \qquad \Rightarrow r^3 = 1728 \qquad \Rightarrow r = 12\ cm$$

Hence, the radius of resulting sphere is 12 cm.

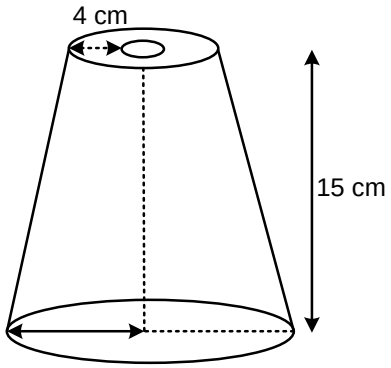
9. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see Fig.). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Sol. Radius of lower part of cap $(r_1) = 10\ cm$

Radius of upper part of cap $(r_2) = 4\ cm$

Slant height of cap = 15 cm

Area of material used for making it = CSA of frustum + Area of upper part



$= \pi(r_1 + r_2)l + \pi r_2$

$= \pi(10+4)\times 15 + \pi \times 4^2$

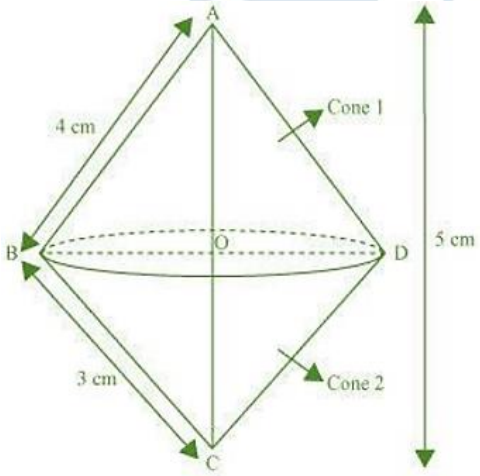
$= 210\pi + 16\pi = 226\pi$

$= 226 \times \frac{22}{7} = 710\frac{2}{7} cm^2$

Hence, the area of material used for making is $710\frac{2}{7} cm^2$.

10. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)

Sol.



The double cone so formed by revolving this right-angled triangle ABC about its hypotenuse is shown in the figure.

Hypotenuse $AC = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\ cm$

Area of ΔABC

$= \frac{1}{2} \times AB \times AC$

$\frac{1}{2} \times AC \times OB = \frac{1}{2} \times 4 \times 3$

$= \frac{1}{2} \times 5 \times OB = 6$

$OB = \frac{12}{5} = 2.4cm$

Volume of double cone = Volume of cone 1 + Volume of cone 2

$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \frac{1}{3}\pi r^2 (h_1 + h_2) = \frac{1}{3}\pi r^2 (\mathrm{OA} + \mathrm{OC})$

$= \frac{1}{3} \times 3.14 \times (2.4)^2 (5)$

$= 30.14\ \mathrm{cm}^3$

Surface area of double cone

= Surface area of cone 1 + Surface area of cone 2

$= \pi r l_1 + \pi r l_2 = \pi r (\ell_1 + \ell_2)$

$= \pi r (4 + 3) = 3.14 \times 2.4 \times 7$

Use small bracket.

$= 52.75\ \mathrm{cm}^2$