NCERT Solutions for Class 10
MATHS – Real Numbers

1. Show that any positive odd integer is of the form $6q+1$ or $6q+3$ or $6q+5$, where q is some integer.

Sol. Let a be any positive integer and b = 6. Then, by Euclid's algorithm, a = 6q + r for some integer $q \geq 0$, and $r = 0,1,2,3,4,5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q+1$ or $6q+2$ or $6q+3$ or $6q+4$ or $6q+5$

Also, $6q+1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q+3 = (6q+2)+1 = 2(3q+1)+1 = 2k_2+1$, where k_2 is an integer

$6q+5 = (6q+4)+1 = 3(3q+2)+1 = 2k_3+1$, where k_3 is an integer

Clearly, $6q+1, 6q+3, 6q+5$ are of the form $2k+1$, where k is an integer.

Therefore $6q+1$, $6q+3, 6q+5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q+1$,

Or $6q+3$,

Or $6q+5$

2. An army contingent of 616 members is to march behind an army band of 32 members is a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. The maximum number of columns would be the HCF of (616, 32)

We can find the HCF of 616 and 32 by using Euclid Division Algorithm.

Therefore,

$$616 = 19 \times 32 + 8$$

$$32 = 4 \times 8 + 0$$

So, HCF of 616 and 32 is 8

Hence, the maximum number of columns in which they can march is 8.

3. Use Euclid's division lemma to show that the square of any positive integer is either of form 3m or 3m + 1 for some integer m.
[**Hint:** Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. How square each of these and show that they can be rewritten in the form 3m or 3m + 1.]

Sol. Let a be any positive integer and b = 3.

Then a = 3q + r for some integer $q \geq 0$

And r = 0, 1, 2 because $0 \leq r < 3$

Therefore, a = 3q or 3q + 1 or 3q + 2

Or,

$$a^2 = (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2$$

$$a^2 = (9q^2) \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4$$

$$= 3 \times (3q^2) \, or \, 3(3q^2 + 2q) + 1 \text{ or } 3(3q^2 + 4q + 1) + 1$$

$$= 3k_1 \, or \, 3k_2 + 1 \, or \, 3k_3 + 1$$

Where k_1, k_2 and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form 3m or 3m + 1.

4. Express each number as product of its prime factors:
(i) 140 (ii) 156 (iii) 3825 (iv) 5005
(v) 7429

Sol. Express each number as a product of its prime factors:

(i) 140

$140 = 2 \times 2 \times 5 \times 7$

$= 2^2 \times 5 \times 7$

(ii) 156

$156 = 2 \times 2 \times 13 \times 3$

$= 2^2 \times 3 \times 13$

(iii) 3825

$3825 = 3 \times 3 \times 5 \times 5 \times 17$

$= 3^2 \times 5^2 \times 7$

(iv) 5005

$5005 = 5 \times 7 \times 11 \times 13$

(v) 7429

Hence, $7429 = 17 \times 19 \times 23$

5. Find the LCM and HCF of the following pairs of integers and verify that $LCM \times HCF =$ product of the two numbers.

(i) 26 and 91

(ii) 510 and 92

(iii) 336 and 54

Sol. **(i) 26 and 91**

$26 = 2 \times 13$

$91 = 7 \times 13$

HCF = 13

$\text{LCM} = 2 \times 7 \times 13 = 182$

Product of the two numbers $= 26 \times 91 = 2366$

$\text{HCF} \times \text{LCM} = 13 \times 182 = 2366$

Hence, product of two numbers $= \text{HCF} \times \text{LCM}$

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

HCF = 2

$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of the two numbers $= 510 \times 92 = 46920$

$\text{HCF} \times \text{LCM} = 2 \times 23460$

= 46920

Hence, product of two numbers $= \text{HCF} \times \text{LCM}$

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$54 = 2 \times 3^3$

$\text{HCF} = 2 \times 3 = 6$

$\text{LCM} = 2^4 \times 3^3 \times 7 = 3024$

Product of the two numbers $= 336 \times 54 = 18144$

$\text{HCF} \times \text{LCM} = 6 \times 3024 = 18144$

Hence, product of two numbers $= \text{HCF} \times \text{LCM}$.

6. Check whether 6^n can end with the digit 0 for any natural number n.

Sol. If the number 6^n ends with the digit zero, then it is divisible by 5.Therefore the prime factorization of 6^n contains the prime 5.This is not possible because the only prime in the factorization of 6^n is 2 and 3 and the uniqueness of the fundamental theorem of arithmetic guarantees that these are no other prime in the factorization of 6^n

Hence, it is very clear that there is no value of n in natural number for which 6^n ends with the digit zero.

7. Prove that $\sqrt{5}$ is irrational.

Sol. Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers $a, b (b \neq 0)$ such that $\sqrt{5} = \frac{a}{b}$

Let a and b have a common factor other than 1. Then we can divide them by the common factor, and assume that a and b are co-prime.

$a = \sqrt{5}b$

$a^2 = 5b^2$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

Let a = 5k, where k is an integer

$(5k)^2 = 5b^2$

$b^2 = 5k^2$. This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

8. Prove that $3 + 2\sqrt{5}$ is irrational.

Sol. Let $3 + 2\sqrt{5}$ is rational.

Therefore, we can find two integers $a, b (b \neq 0)$ such that

$3 + 2\sqrt{5} = \frac{a}{b}$

$2\sqrt{5} = \frac{a}{b} - 3$

$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$

Since a and b are integers, $\frac{1}{2}\left(\frac{a}{b}-3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false.

Therefore, $3+2\sqrt{5}$ is irrational.

9. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Sol. **(i)** $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455 = 5 \times 7 \times 13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600} = \frac{3}{320}$

$320 = 2^6 \times 5$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$343 = 7^3$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 5^2}$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

(vii) $\frac{129}{2^2 5^7 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15}$

$\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$

The denominator is of the form 5^n.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50}$

$\frac{35}{50} = \frac{7 \times 5}{10 \times 5} = \frac{7}{10}$

$10 = 2 \times 5$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210}$

$\frac{77}{210} = \frac{11 \times 7}{30 \times 7} = \frac{11}{30}$

$30 = 2 \times 3 \times 5$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 3 as its factors, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

10. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000…

(iii) $43.\overline{123456789}$

Sol. **(i) 43.123456789**

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000…

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating recurring the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$