1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) x^2-2x-8 (ii) $4s^2-4s+1$ (iii) $6x^2-3-7x$ (iv) $4u^2+8u$

(v) t^2-15 (vi) $3x^2-x-4$

Sol. **(i)** $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when $x-4=0$ or $x+2=0$, i.e. when $x=4$ or $x=-2$

Therefore, the zeroes of x^2-2x-8 and 4 and – 2.

Sum of zeroes $=4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1$

The value of $4s^2-4s+1$ is zero when $2s-1=0$, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2-4s+1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes $=\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of s})}{\text{Coefficient of s}^2}$

Product of zeroes $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of s}^2}$

(iii) $6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2-3-7x$ is zero when $3x+1=0$ or $2x-3=0$, i.e., $x=\frac{-1}{3}$ or $x=\frac{3}{2}$

Therefore, the zeroes of $6x^2-3-7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$

Sum of zeroes $=\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{-1}{3}\times\frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2+8u=4u^2+8u+0$

$=4u(u+2)$

The value of $4u^2+8u$ is zero when $4u=0$ or $u+2=0$, i.e., $u=0$ or $u=-2$.

Therefore, the zeroes of $4u^2+8u$ are 0 and – 2.

Sum of zeroes $=0+(-2)=-2=\frac{-(+8)}{4}=\frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes $=0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v) $t^2-15=(t-\sqrt{15})\ (t+\sqrt{15})$

The value of t^2-15 is zero

When $t=+\sqrt{15}$ or $-\sqrt{15}$

Therefore, the zeroes of

t^2-15 are $\pm\sqrt{15}$.

Sum of zeroes $=+\sqrt{15}+(-\sqrt{15})$

$=0=\frac{0}{1}=\frac{-(\text{coefficient of t})}{\text{coefficient of } t^2}$

Product of zeroes

$=(+\sqrt{15})\times(-\sqrt{15})$

$=-15$

$=\frac{-15}{1}=\frac{\text{constant}}{\text{coefficient of } t^2}$

(vi) $3x^2 - x - 4 = (3x-4)(x+1)$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e.,

When $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and – 1.

Sum of zeroes $= \frac{4}{3} + (-1) = \frac{1}{3}$

$= \frac{-(-1)}{3} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

Product of zeroes $= \frac{4}{3} \times (-1) = \frac{-4}{3}$

$= \frac{\text{constant term}}{\text{coefficient of } x^2}$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$ (iii) $0, \sqrt{5}$ (iv) 1, 1

(v) $-\frac{1}{4}, \frac{1}{4}$ (vi) 4,1

Sol. **(i)** $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$

$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$

If a = 4, then $b = -1, c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If a = 3, then $b = -3\sqrt{2}, c = 1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}\,x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

If a = 1, then $b = 0, c = \sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1, 1

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If a = 1, then $b = -1, c = 1$

Therefore, the quadratic polynomial is $x^2 - x + 1$.

(v) $-\frac{1}{4}, \frac{1}{4}$

Let the polynomial be $ax^2 + bx + c,$ and its zeroes be α and β.

$\alpha + \beta = \frac{-1}{4} = \frac{-b}{a}$

$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$

If $a = 4,$ then $b = 1, c = 1$

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

(vi) 4, 1

$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$

$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$

If a = 1, then $b = -4, c = 1$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

3. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

Sol. **(i)**

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 \qquad -6t^2 \\
 & - \qquad\quad + \\
\hline
 & 2t^3 + 4t^2 - 9t - 12 \\
 & 3t^3 \qquad -9t \\
 & - \qquad\quad + \\
\hline
 & 4t^2 \qquad - 12 \\
 & 4t^2 \qquad - 12 \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & 3x^4+9x^3+3x^2 \\
 & -\quad -\quad - \\
\hline
 & -4x^3-10x^2+2x+2 \\
 & -4x^3-12x^2-4x \\
 & +\quad +\quad + \\
\hline
 & \qquad 2x^2+6x+2 \\
 & \qquad 2x^2+6x+2 \\
 & \qquad -\quad -\quad - \\
\hline
 & \qquad 0 \\
\hline
\end{array}
$$

Since the remainder is 0.

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

$$
\begin{array}{r|l}
 & x^2-1 \\
\hline
x^2+3x+1 & x^5-4x^3+x^2+3x+1 \\
 & x^5-3x^3+x^2 \\
 & -\quad +\quad - \\
\hline
 & -x^3\qquad +3x+1 \\
 & -x^3\qquad +3x-1 \\
 & +\qquad -\quad + \\
\hline
 & \qquad\qquad 2 \\
\hline
\end{array}
$$

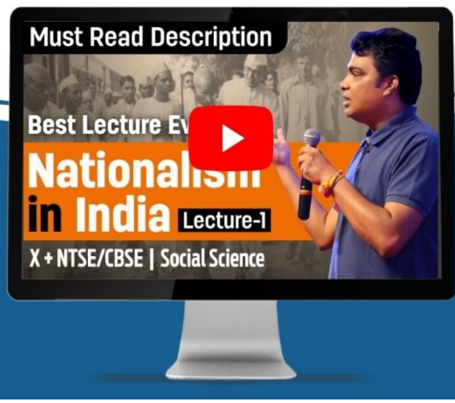
Since the remainder $\neq 0$,

x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$

4. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. $p(x)=3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,



$\therefore \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
\hline
x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\
\hline
 & \qquad 6x^3+3x^2-10x-5 \\
 & \qquad 6x^3+0x^2-10x \\
 & \qquad -\quad -\quad + \\
\hline
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad -\quad -\quad + \\
\hline
 & \qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

$3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)(3x^2+6x+3)$

$=3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$

$\Rightarrow p(x)=3\left(x^2-\frac{5}{3}\right)(x+1)^2$

We factorize x^2+2x+1

Therefore, its zero is given by $x+1=0$

$x=-1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x=-1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}},-\sqrt{\frac{5}{3}},-1$ and -1.

5. On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$, respectively. Find $g(x)$.

Sol. $p(x)=x^3-3x^2+x+2$ (Dividend)

$g(x)=?$ (Divisor)

Quotient $=(x-2)$

Remainder $=(-2x+4)$

Dividend = Divisor $\times$ Quotient + Remainder

$x^3-3x^2+x+2=g(x)\times(x-2)+(-2x+4)$

$x^3-3x^2+x+2+2x-4=g(x)(x-2)$

$x^3-3x^2+3x-2=g(x)(x-2)$

$g(x)$ is the quotient when we divide (x^3-3x^2+3x-2) by $(x-2)$

$$
\begin{array}{r|l}
 & x^2-x+1 \\
\hline
x-2 & x^3-3x^2+3x-2 \\
 & x^3-2x^2 \\
 & -\quad + \\
\hline
 & -x^2+3x-2 \\
 & -x^2+2x \\
 & +\quad - \\
\hline
 & x-2 \\
 & x-2 \\
 & -\quad + \\
\hline
 & 0
\end{array}
$$

$\therefore g(x)=(x^2-x+1)$

6. Give examples of polynomials $p(x)$, $g(x)$, q(x) and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x)=\deg q(x)$ (ii) $\deg q(x)=\deg r(x)$ (iii) $\deg r(x)=0$

Sol. According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x)\neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x)=g(x)\times q(x)+r(x)$, Degree of $r(x)=0$ or degree of $r(x)<$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)=\deg q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2+2x+2$ by 2.

Here, $p(x)=6x^2+2x+2$

$g(x)=2$

$q(x)=3x^2+x+1$ and $r(x)=0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x)=g(x)\times q(x)+r(x)$

$6x^2+2x+2=2(3x^2+x+1)$

Thus, the division algorithm is satisfied.

(ii) $\deg q(x)=\deg r(x)$

Let us assume the division of x^3+x by x^2,

Here, $p(x)=x^3+x$

$g(x)=x^2$

$q(x)=x$ and $r(x)=x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(\mathrm{x})=\mathrm{g}(\mathrm{x})\times q(x)+\mathrm{r}(\mathrm{x})$

$x^3+x=(x^2)\times x+x$

$x^3+x=x^3+x$

Thus, the division algorithm is satisfied.

(iii) deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$ by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^2 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

7. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Sol. Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta,$ and γ.

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If a = 1, then $b = -2$, $c = -7$, $d = 14$

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

8. If the zeroes of the polynomial x^3-3x^2+x+1 are $a-b,\ a,\ a+b,$ find a and b.

Sol. $p(x)=x^3-3x^2+x+1$

Zeroes are $a-b,\ a$ and $a+b$

Comparing the given polynomial with $px^3+qx^2+rx+t,$ we obtain

$p=1,\ q=-3,\ r=1,\ t=1$

Sum of zeroes $=a-b+a+a+b$

$\frac{-q}{p}=3a$

$\frac{-(-3)}{1}=3a$

$3=3a$

$a=1$

The zeroes are $1-b, 1, 1+b$.

Multiplication of zeroes $=(1-b)(1+b)$

$\frac{-t}{p}=1-b^2$

$=\frac{-1}{1}=1-b^2$

$1-b^2=-1$

$1+1=b^2$

$b=\pm\sqrt{2}$

Hence, a = 1 and $b=\sqrt{2}$ or $-\sqrt{2}$.

9. If two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3},$ find other zeroes.

Sol. Given that $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right)=x^2+4-4x-3=x^2-4x+1$ is a factor of the given polynomial.

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4-6x^3-26x^2+138x-35$ by x^2-4x+1.

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+x^2 \\
 & - \quad + \qquad - \\
\hline
 & -2x^3-27x^2+138x-35 \\
 & -2x^3-8x^2-2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Clearly, $x^4-6x^3-26x^2+138x-35=(x^2-4x+1)(x^2-2x-35)$

It can be observed that $\left(x^2-2x-35\right)$ is also a factor of the given polynomial.

And $\left(x^2-2x-35\right)=(x-7)(x+5)$

Therefore, the value of the polynomial is also zero when $x=7$ or -5

Hence, 7 and -5 are also zeroes of this polynomial.

10. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x+a$, find k and a.

Sol. By division algorithm,

Dividend = divisor $\times$ Quotient + Remainder

Dividend – Remainder = Divisor $\times$ Quotient

$x^4-6x^3+16x^2-25x+10-x-a=x^4-6x^3+16x^2-26x+10-a$ will be perfectly divisible by x^2-2x+k.

Let us divide $x^4-6x^3+16x^2-26x+10-a$ by x^2-2x+k

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-26x+10-a \\
 & x^4-2x^3+kx^2 \\
 & -\quad+\qquad- \\
\hline
 & -4x^3+(16-k)x^2-26x \\
 & -4x^3+\qquad 8x^2-4kx \\
 & +\quad-\qquad\quad+ \\
\hline
 & (8-k)x^2-(26-4k)x+10-a \\
 & (8-k)x^2-(16-2k)x+(8k-k^2) \\
 & -\qquad\quad+\qquad\quad- \\
\hline
 & (-10+2k)x+(10-a-8k+k^2)
\end{array}
$$

It can be observed that $(-10+2k)x+(10-a-8k+k^2)$ will be 0.

On comparing.

$\Rightarrow \qquad -10+2k=0$

$\Rightarrow \qquad \boxed{k=5}$

$10-a-8k+k^2=0 \qquad$ (Put $\quad k=5$)

$\Rightarrow \qquad 10-a-8(5)+5^2=0$

$\Rightarrow \qquad \boxed{a=-5}$

Hence $k=5$ and $a=-5$