1. Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$ (ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$ (iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$ (vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$ (viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Sol. **(i)** $(x+1)^2 = 2(x-3) \Rightarrow x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3-x) \Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3) \Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow 3x - 1 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5) \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x \Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1) \Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5 \Rightarrow x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x-2)^2 \Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x \Rightarrow 7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(vii) $(x+2)^3 = 2x(x^2-1) \Rightarrow x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x \Rightarrow x^3 - 14x - 6x^2 - 8 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(viii) $x^3 - 4x^3 - x + 1 = (x-2)^3 \Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x \Rightarrow 2x^2 - 13x + 9 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

2. Solve the problems given in Example 1.

Sol. The problems given in the example 1 are $x^2-45x+324=0$ and $x^2-55x+750=0$

(a) $x^2-45x+324=0$

$x^2-36x-9x+324=0$

$\Rightarrow x(x-36)-9(x-36)=0$

$\Rightarrow (x-9)(x-36)=0$

Either $x=9$ or 36

Hence John and Jivanti have 36 and 9 marbles respectively in the beginning

(b) $x^2-55x+750=0$

$\Rightarrow x^2-30x-25x+750=0$

$\Rightarrow x(x-30)-25(x-30)=0$

$\Rightarrow (x-30)(x-25)=0$

Either $x=30$ or 25

Hence number of toys on that day was 30 or 25.

3. Find two numbers whose sum is 27 and product is 182.

Sol. Let the first number be x and the second number is 27 – x.

Therefore, their product $=x(27-x)$

It is given that the product of these numbers is 182.

Therefore, $x(27-x)=182$

$\Rightarrow x^2-27x+182=0$

$\Rightarrow x^2-13x-14x+182=0$

$\Rightarrow x(x-13)-14(x-13)=0$

$\Rightarrow (x-13)(x-14)=0$

Either $x-13=0$ or $x-14=0$

i.e., $x=13$ or $x=14$

If first number = 13, then

Other number = 27 – 13 = 14

If first number = 14, then

Other number = 27 – 14 = 13

Therefore, the numbers are 13 and 14.

4. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2-7x+3=0$ (ii) $2x^2+x-4=0$

(iii) $4x^2+4\sqrt{3}x+3=0$ (iv) $2x^2+x+4=0$

Sol. **(i)** $2x^2 - 7x + 3 = 0$

$\Rightarrow 2x^2 - 7x = -3$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$

On adding $\left(\frac{7}{4}\right)^2$ to both sides of equation, we obtain

$\Rightarrow (x)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow \left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$

$\Rightarrow x = \frac{7}{4} \pm \frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = \frac{12}{4}$ or $x = \frac{2}{4}$

$\Rightarrow x = 3$ or $\frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$

$\Rightarrow 2x^2 + x = 4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = 2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

$\Rightarrow x + \frac{1}{4} = \pm \frac{\sqrt{33}}{4}$

$\Rightarrow x = \pm \frac{\sqrt{33}}{4} - \frac{1}{4}$

$\Rightarrow x = \frac{\pm\sqrt{33} - 1}{4}$

$\Rightarrow x = \frac{\sqrt{33} - 1}{4}$ or $\frac{-\sqrt{33} - 1}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + \left(\sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right) = 0$ and $\left(2x + \sqrt{3}\right) = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{2}$ and $x = \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

$\Rightarrow 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = -2$

$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} = -2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2 = -\frac{31}{16}$$

However, the square of a number cannot be negative.
Therefore, there is no real root for the given equation.

5. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Sol. $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -7, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{25}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

$$\Rightarrow x = \frac{7+5}{4} \text{ or } \frac{7-5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 3 \text{ or } \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 32}}{4}$$

$$x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\therefore x = \frac{-1 + \sqrt{33}}{4} \text{ or } \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2+4\sqrt{3}x+3=0$

On comparing this equation with $ax^2+bx+c=0,$ we obtain

$a=4, b=4\sqrt{3}, c=3$

By using quadratic formula, we obtain

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-4\sqrt{3}\pm\sqrt{48-48}}{8}$$

$$\Rightarrow x=\frac{-4\sqrt{3}+0}{8}$$

$$\therefore x=\frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

(iv) $2x^2+x+4=0$

On comparing this equation with $ax^2+bx+c=0,$ we obtain

$a=2, b=1, c=4$

By using quadratic formula, we obtain

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{1-32}}{4}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{-31}}{4}$$

However, the square of a number cannot be negative.
Therefore, there is no real root for the given equation.

6. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol. Let the time taken by larger tap to fill the tank $=x$ hours

So, the time taken by smaller tap to fill the tank $=(x+10)$ hours.

Therefore, in one hour, tank filled by larger tap $=\frac{1}{x}$

Therefore, in one hour, tank filled by smaller tap $=\frac{1}{x+10}$

According to question

$$\frac{1}{x}+\frac{1}{x+10}=\frac{1}{9\frac{3}{8}}$$

$$\Rightarrow \quad 75(2x+10)=8x(x+10)$$

$$\Rightarrow \quad 4x^2-35x-375=0$$

$$\Rightarrow \quad (x-15)(4x+25)=0$$

$$\Rightarrow \quad x=(15) \text{ or } \left(\frac{-25}{4}\right)$$

But time to fill the tank can't be negative.

$\therefore$ time taken by larger tap to fill the tank = 15 hours

Time taken by smaller tap to fill the tank $=15+10=25$ hours.

7. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

Sol. Let the average speed of passenger train be x km/h.

Average speed of express train $=(x+11)$km/h

It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x}-\frac{132}{x+11}=1$$

$$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right]=1$$

$$\Rightarrow \frac{132\times 11}{x(x+11)}=1$$

$$\Rightarrow 132\times 11=x(x+11)$$

$$\Rightarrow x^2+11x-1452=0$$

$$\Rightarrow x^2+44x-33x-1452=0$$

$$\Rightarrow x(x+44)-33(x+44)=0$$

$$\Rightarrow (x+44)(x-33)=0$$

$$\Rightarrow x=-44,33$$

Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

8. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2-3x+5=0$ (ii) $3x^2-4\sqrt{3}x+4=0$ (iii) $2x^2-6x+3=0$

Sol. We know that for a quadratic equation $ax^2+bx+c=0$, discriminant is b^2-4ac.

(A) If $b^2-4ac>0\rightarrow$ two distinct real roots

(B) If $b^2-4ac=0\rightarrow$ two equal real roots

(C) If $b^2-4ac<0\rightarrow$ no real roots

(I) $2x^2-3x+5=0$

Comparing this equation with $ax^2+bx+c=0$, we obtain $a=2, b=-3, c=5$

Discriminant $=b^2-4ac=(-3)^2-4(2)(5)=9-40$

$=-31$

As $b^2-4ac<0$,

Therefore, no real roots is possible for the given equation.

(II) $3x^2-4\sqrt{3}x+4=0$

Comparing this equation with $ax^2+bx+c=0$, we obtain

$a=3, b=-4\sqrt{3}, c=4$

Discriminant $=b^2-4ac=\left(-4\sqrt{3}\right)^2-4(3)(4)$

$=48-48=0$

As $b^2-4ac=0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$.

Therefore, the roots are $\frac{2\sqrt{3}}{3}$ and $\frac{2\sqrt{3}}{3}$

(III) $2x^2-6x+3=0$

Comparing this equation with $ax^2+bx+c=0$, we obtain

$a=2, b=-6, c=3$

Discriminant $=b^2-4ac=(-6)^2-4\ (2)\ (3)$

$=36-24=12$

As $b^2-4ac>0$,

Therefore, distinct real roots exist for this equation as follows.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

9. Find the values of k for each of the following quadratic equations, so that they have two equal roots.
(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x-2) + 6 = 0$

Sol. We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant $(b^2 - 4ac)$ will be 0.

(i) $2x^2 + kx + 3 = 0$

Comparing equation with $ax^2 + bx + c = 0$ we obtain
$a = 2, b = k, c = 3$

Discriminant $= b^2 - 4ac = (k)^2 - 4(2)(3)$
$= k^2 - 24$
For equal roots,
Discriminant = 0
$k^2 - 24 = 0$
$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(II) $kx(x-2) + 6 = 0$
Or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx = c = 0,$ we obtain
$a = k, b = -2k, c = 6$

Discriminant $= b^2 - 4ac = (-2k)^2 - 4(k)(6)$
$= 4k^2 - 24k$
For equal roots,
$b^2 - 4ac = 0$
$4k^2 - 24k = 0$
$4k(k-6) = 0$
Either 4k = 0 or k = 6 = 0
k = 0 or k = 6

However, if k = 0, then the equation will not have the terms $'x^2'$ and 'x'.
Therefore, if this equation has two equal roots, k should be 6 only.

10. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800\,m^2$? If so, find its length and breadth.

Sol. Let the breadth of mango grove be l.

Length of mango grove will be $2l$.

Area of mango grove $=(2l)(l)$

$=2l^2$

$2l^2=800$

$l^2=\frac{800}{2}=400$

$l^2-400=0$

Comparing this equation with $al^2+bl+c=0,$ we obtain

$a=1, b=0, c=400$

Discriminant $=b^2-4ac=(0)^2-4\times(1)\times(-400)=1600$

Here, $b^2-4ac>0$

Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.

$l=\pm 20$

However, breadth cannot be negative.

Therefore, breadth of mango grove = 20 m

Length of mango grove $=2\times 20=40m$