1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs. 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs. 150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest at 8 % per annum.

Sol. **(i)** Given: Fare of First Km = Rs.15

Fare for each Additional Km = Rs.8

Hence, Taxi fare for 1st km = 15

Taxi fare for first 2 km = 15 + 8 = 23

Taxi fare for first 3 km = 23 + 8 = 31

Taxi fare for first 4 km = 31 + 8 = 39

Hence, the Series formed is 15, 23, 31, 39

Since here the terms continuously increases by the same number 8, the above list forms an A.P.

(ii) Let the initial volume of air in a cylinder be V lit. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time.

In other words, after every stroke, only $1-\frac{1}{4}=\frac{3}{4}$th part of air will remain.

Hence the series can be written as below

$$v, \frac{3v}{4}, \frac{3v}{4}-\left(\frac{3v}{4}\right)\left(\frac{1}{4}\right)....$$

$$= v, \frac{3v}{4}, \frac{12v-3v}{16}...$$

$$= v, \frac{3v}{4}, \frac{9v}{16}$$

Clearly, it can be observed that the adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

(iii) Cost of digging for first metre = 150
Cost of digging for first 2 metres = 150 + 50 = 200
Cost of digging for first 3 metres = 200 + 50 = 250
Cost of digging for first 4 metres = 250 + 50 = 300
Clearly, 150, 200, 250, 300… forms an A.P. because every term is 50 more than the preceding term.

(iv) We know that if Rs P is deposited at r% compound interest per annum for n years, our money will be $P\left(1+\frac{r}{100}\right)^n$ after n years.

Therefore, after every year, our money will be

$$10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^2, 10000\left(1+\frac{8}{100}\right)^3, 10000\left(1+\frac{8}{100}\right)^4, ...$$

Cleary, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

2. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the AP:

	a	d	n	a_n
(i)	7	3	8	…
(ii)	-8	…	10	0
(iii)	…	-3	18	-5
(iv)	-18.9	2.5	…	3.6
(v)	3.5	0	105	…

Sol. **(i)** a = 7, d = 3, n = 8, $a_n = ?$

We know that,
For an A.P. $a_n = a+(n-1)d$

$= 7+(8-1)3$

$= 7+(7)\ 3$

$= 7+21 = 28$

Hence, $a_n = 28$

(ii) Given that

$a=-18,\ n=10,\ a_n=0,\ d=?$

We know that,

$a_n = a+(n-1)d$

$0 = -18+(10-1)d$

$18 = 9d$

$d = \frac{18}{9} = 2$

Hence, common difference, $d = 2$

(iii) Given that $d = -3,\ n = 18,\ a_n = -5$

We know that,

$a_n = a + (n-1)d$

$-5 = a + (18-1)(-3)$

$-5 = a + (17)(-3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

Hence, a = 46

(iv) $a = -18.9,\ d = 2.5,\ a_n = 3.6, n = ?$

We know that,

$a_n = a + (n-1)d$

$3.6 = -18.9 + (n-1)2.5$

$3.6 + 18.9 = (n-1)2.5$

$22.5 = (n-1)2.5$

$(n-1) = \dfrac{22.5}{2.5}$

$n - 1 = 9$

$n = 10$

Hence, $n = 10$

(v) $a = 3.5,\ d = 0, n = 105,\ a_n = ?$

We know that,

$a_n = a + (n-1)d$

$a_n = 3.5 + (105-1)0$

$a_n = 3.5 + 104 \times 0$

$a_n = 3.5$

Hence $a_n = 3.5$

3. Choose the correct choice in the following and justify :

(i) 30th term of the AP: 10, 7, 4, . . . , is

(A) 97 (B) 77 (C) –77 (D) – 87

(ii) 11th term of the AP : $-3, -\dfrac{1}{2}, 2, \ldots,$

(A) 28 (B) 22 (C) –38 (D) $-48\dfrac{1}{2}$

Sol. **(i)** Given that

A.P. 10, 7, 4, …

First term, a = 10

Common difference, $d = a_2 - a_1 = 7 - 10$

$= -3$

We know that $a_n = a + (\text{n}-1)\text{d}$

$a_{30} = 10 + (30-1)(-3)$

$a_{30} = 10 + (29)(-3)$

$a_{30} = 10 - 87 = -77$

Hence, the correct answer is C.

(ii) Given that A.P.

First term $a = -3$

Common difference, $d = a_2 - a_1$

$= -\frac{1}{2} - (-3)$

$= -\frac{1}{2} + 3 = \frac{5}{2}$

We know that,

$a_n = a + (n-1)d$

$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$

$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$

$a_{11} = -3 + 25$

$a_{11} = 22$

Hence, the answer is B.

4. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

Sol. 3, 8, 13, 18, …

For this A.P.,

$a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let nth term of this A.P. be 78.

$a_n = a + (n-1)d$

$78 = 3 + (n-1)5$

$75 = (n-1)5$

$(n-1) = 15$

$n = 16$

5. Check whether – 150 is a term of the AP : 11, 8, 5, 2 ...

Sol. For this A.P.

$a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

Let -150 **be the nth term of this A.P.**

We know that,

$a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-164 = -3n$

$n = \frac{164}{3}$

Clearly, n is not an integer.

Therefore, - 150 is not a term of this A.P.

6. In an AP:

(i) Given $a = 5,\ d = 3,\ a_n = 50$, n and S_n.

(ii) Given $a = 7,\ a_{13} = 35,$ find d and S_{13}.

(iii) Given $a_{12} = 37, d = 3,$ find a and S_{12}.

(iv) Given $a_3 = 15,\ S_{10} = 125,$ find d and a_{10}.

(v) Given $d = 5,\ S_9 = 75,$ find a and a_9.

(vi) Given $a = 2,\ d = 8,\ S_n = 90,$ find n and a_n.

(vii) Given $a = 8,\ a_n = 62,\ S_n = 210,$ find n and d.

(viii) Given $a_n = 4,\ d = 2,\ S_n = -14,$ find n and a.

(ix) Given $a = 3,\ n = 8,\ S = 192$ find d.

(x) Given $l = 28,\ S = 144,$ and there are total 9 terms. Find a.

Sol. **(i)** Given that, a = 5, d = 3, $a_n = 50$

As $a_n = a + (\mathrm{n}-1)\mathrm{d}$,

$\therefore 50 = 5 + (n-1)3$

$45 = (n-1)3$
$15 = n-1$
$n = 16$

$S_n = \frac{n}{2}[a + a_n]$

$S_{16} = \frac{16}{2}[5+50]$

$= 8 \times 55$
$= 440$

(ii) Given that, $a = 7,\ a_{13} = 35$
As $a_n = a + (n-1)d,$
$\therefore a_{13} = a + (13-1)d$
$35 = 7 + 12d$
$35 - 7 = 12d$
$28 = 12d$

$d = \frac{7}{3}$

$S_n = \frac{n}{2}[a + a_n]$

$S_{13} = \frac{n}{2}[a + a_{13}]$

$= \frac{13}{2}[7+35]$

$= \frac{13 \times 42}{2} = 13 \times 21$

$= 273$

(iii) Given that, $a_{12} = 37,\ d = 3$
As $a_n = a + (n-1)d,$
$a_{12} = a + (12-1)3$
$37 = a + 33$
$a = 4$

$S_n = \frac{n}{2}[a + a_n]$

$S_{12} = \frac{12}{2}[4+37]$

$S_{12} = 6(41)$
$S_{12} = 246$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

As $a_n = a + (n-1)d$,

$a_3 = a + (3-1)d$

$15 = a + 2d$ (i)

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{10} = \frac{10}{2}\left[2a + (10-1)d\right]$

$125 = 5(2a + 9d)$

$25 = 2a + 9d \qquad \ldots\text{(ii)}$

On multiplying equation (1) by 2, we obtain

$30 = 2a + 4d \qquad \ldots\text{(iii)}$

On subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (i),

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10-1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v) Given that, $d = 5$, $S_9 = 75$

As, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$,

$S_9 = \frac{9}{2}\left[2a + (9-1)5\right]$

$75 = \frac{9}{2}(2a + 40)$

$25 = 3(a + 20)$

$25 = 3a + 60$

$3a = 25 - 60$

$a = \frac{-35}{3}$

$a_n = a + (n-1)d$

$a_9 = a + (9-1)\ (5)$

$= \dfrac{-35}{3} + 8\ (5)$

$= \dfrac{-35}{3} + 40$

$= \dfrac{-35+120}{3} = \dfrac{85}{3}$

(iv) Given that, $a = 2,\ d = 8,\ S_n = 90$

As $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$,

$90 = \dfrac{n}{2}\left[4 + (n-1)8\right]$

$90 = n\left[2 + (n-1)4\right]$

$90 = n[2 + 4n - 4]$

$90 = n(4n-2) = 4n^2 - 2n$

$4n^2 - 2n - 90 = 0$

$4n^2 - 20n + 18n - 90 = 0$

$4n(n-5) + 18(n-5) = 0$

$(n-5)(4n+18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$n = 5$ or $n = -\dfrac{18}{4} = \dfrac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

$a_n = a + (n-1)d$

$a_5 = 2 + (5-1)8$

$= 2 + (4)\ (8)$

$= 2 + 32 = 34$

(vii) Given that, $a = 8,\ a_n = 62,\ S_n = 210$

$S_n = \dfrac{n}{2}\left[a + a_n\right]$

$210 = \dfrac{n}{2}\left[8 + 62\right]$

$210 = \dfrac{n}{2}(70)$

$n = 6$

$a_n = a + (n-1)d$

$62 = 8 + (6-1)d$

$62 - 8 = 5d$

$54 = 5d$

$d = \frac{54}{5}$

(viii) Given that, $a_n = 4,\ d = 2,\ S_n = -14$

$a_n = a + (n-1)d$

$4 = a + (n-1)2$

$4 = a + 2n - 2$

$a + 2n = 6$

$a = 6 - 2n$ (i)

$S_n = \frac{n}{2}[a + a_n]$

$-14 = \frac{n}{2}[a + 4]$

$-28 = n(a+4)$

$-28 = n(6 - 2n + 4)$ {From equation (i)}

$-28 = n(-2n + 10)$

$-28 = -2n^2 + 10n$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n-7) + 2(n-7) = 0$

$(n-7)(n+2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$

$n = 7$ or $n = -2$

However, n can neither be negative nor fractional.

Therefore, n = 7

From equation (i), we obtain

$a = 6 - 2n$

$a = 6 - 2(7)$

$= 6 - 14$

$= -8$

(ix) Given that, $a = 3,\ n = 8,\ S = 192$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$192 = \frac{8}{2}[2 \times 3 + (8-1)\,\mathrm{d}]$$

$$192 = 4\left[6 + 7d\right]$$

$$48 = 6 + 7d$$

$$42 = 7d$$

$$d = 6$$

(x) Given that, $l = 28, S = 144$ and there are total of 9 terms.

$$S_n = \frac{n}{2}(a + l)$$

$$144 = \frac{9}{2}(a + 28)$$

$$(16) \times (2) = a + 28$$

$$32 = a + 28$$

$$a = 4$$

7. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

Sol. Let there be n terms of this A.P.

For this A.P., a = 9

$$d = a_2 - a_1 = 17 - 9 = 8$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$636 = \frac{n}{2}\left[2 \times a + (n-1)8\right]$$

$$636 = \frac{n}{2}\left[18 + (n-1)8\right]$$

$$636 = n\left[9 + 4n - 4\right]$$

$$636 = n\,(4n + 5)$$

$$4n^2 + 5n - 636 = 0$$

$$4n^2 + 53n - 48n - 636 = 0$$

$$n(4n + 53) - 12(4n + 53) = 0$$

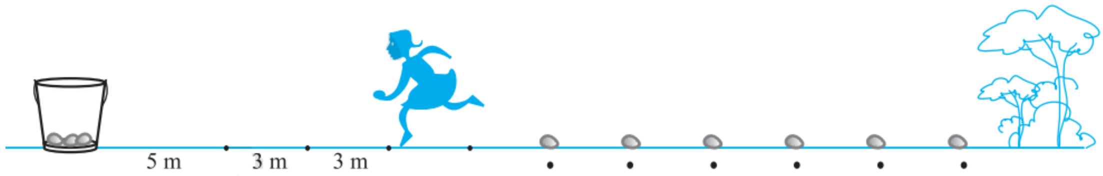
$$(4n + 53)(n - 12) = 0$$

Either $4n + 53 = 0$ or $n - 12 = 0$

$n = \frac{-53}{4}$ or $n = 12$

n cannot be $-\frac{53}{4}$. As the number of terms can neither be negative nor fractional, therefore, $n = 12$ only.

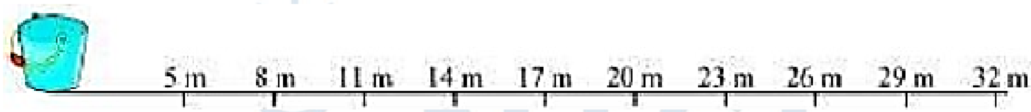
8. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[**Hint:** To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Sol.



The distances of potatoes are as follows.

5, 8, 11, 14…

It can be observed that these distances are in A.P.

$a = 5$

$d = 8 - 5 = 3$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(5) + (10-1)3]$

$= 5[10 + 9 \times 3]$

$= 5(10 + 27) = 5(37)$

= 185

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.

Therefore, total distance that the competitor will run $= 2 \times 185$

= 370 m

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14…

Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept. Therefore, distances to be run are

10, 16, 22, 28, 34,……..

$a = 10$

$d = 16 - 10 = 6$

$S_{10} = ?$

$S_{10} = \frac{10}{2}[2\times 10 + (10-1)6]$

$= 5[20+54]$

$= 5(74)$

$= 370$

Therefore, the competitor will run a total distance of 370 m.

9. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Sol. We know that,

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d$

$a_3 = a + 2d$

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$(a+2d) + (a+6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 - 4d$ (i)

Also, it is given that $(a_3)\times(a_7) = 8$

$(a+2d)\times(a+6d) = 8$

From equation (i),

$(3-4d+2d)\times(3-4d+6d) = 8$

$(3-2d)\times(3+2d) = 8$

$9 - 4d^2 = 8$

$4d^2 = 9 - 8 = 1$

$d^2 = \frac{1}{4}$

$d = \pm\frac{1}{2}$

$d = \frac{1}{2}$ or $-\frac{1}{2}$

From equation (i),

$\left(\text{When d is } \frac{1}{2}\right)$

$a = 3 - 4d$

$a = 3 - 4\left(\frac{1}{2}\right)$

$= 3 - 2 = 1$

$\left(\text{When d is} -\frac{1}{2}\right)$

$a = 3 - 4\left(-\frac{1}{2}\right)$

$a = 3 + 2 = 5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\left(\text{When a is 1 and d is } \frac{1}{2}\right)$

$S_{16} = 8\left[2 + \frac{15}{2}\right]$

$= 4(19) = 76$

$\left(\text{When a is 5 and d is} -\frac{1}{2}\right)$

$S_{16} = \frac{16}{2}\left[2(5) + (16-1)\left(-\frac{1}{2}\right)\right]$

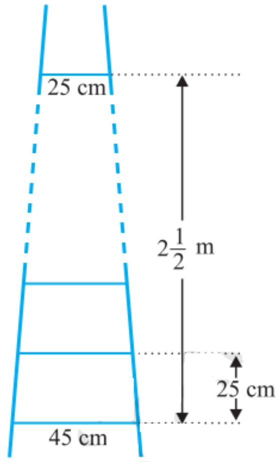
$= 8\left[10 + (15)\left(-\frac{1}{2}\right)\right]$

$= 8\left(\frac{5}{2}\right)$

$= 20$

10. A ladder has rungs 25 cm apart. (see Fig.). The rungs decrease uniformly in length from 5 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[**Hint:** Number of rungs $=\frac{250}{25}+1$]



Sol. It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

$\therefore$ Total number of rungs $=\dfrac{2\frac{1}{2}\times 100}{25}+1=\dfrac{250}{25}+1=11$

Now, as the lengths of the rungs decrease uniformly, they will be in an A. P.

The length of the wood required for the rungs equals the sum of all the terms of this A.P.
First term, a = 45
Last term, l = 25
n = 11

$S_n=\frac{n}{2}(a+l)$

$\therefore S_{10}=\frac{11}{2}(45+25)=\frac{11}{2}(70)=385$ cm

Therefore, the length of the wood required for the rungs is 385 cm.