1. Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.

Sol. A line segment of length 7.6 cm can be divided in the ratio of 5 : 8 as follows.

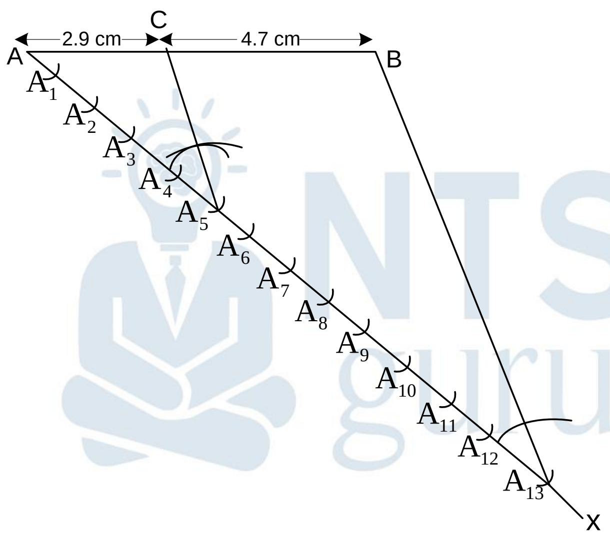
Step 1 Draw line segment AB of 7.6 cm and draw a ray AX making an acute angle with segment AB.

Step 2 Locate 13 (= 5 + 8) points, $\mathrm{A_1, A_2, A_3, A_4 .... A_{13}}$, on AX such that $AA_1 = \mathrm{A_1A_2 = A_2A_3}$ and so on.

Step 3 Join $\mathrm{BA_{13}}$

Step 4 Through the point $\mathrm{A_5}$, draw a line parallel to $\mathrm{BA_{13}}$ (by making an angle equal to $\angle \mathrm{AA_{13}B}$) at $\mathrm{A_5}$ intersecting AB at point C.

C is the point dividing line segment AB of 7.6 cm in the required ratio of 5 : 8. The lengths of AC and CB can be measured. It comes out to 2.9 cm and 4.7 cm respectively.



Justification

The construction can be justified by proving that

$$\frac{AC}{CB} = \frac{5}{8}$$

By construction, we have $A_5C \parallel A_{13}B$. By applying Basic proportionality theorem for the triangle $AA_{13}B$, we obtain

$$\frac{AC}{CB} = \frac{AA_5}{A_5A_{13}} \qquad \ldots(1)$$

From the figure, it can be observed that AA_5 and A_5A_{13} obtain 5 and 8 equal divisions of line segments respectively.

$\therefore \dfrac{AA_5}{A_5A_{13}} = \dfrac{5}{8}$ …(2)

On comparing equations (1) and (2), we obtain

$\dfrac{AC}{CB} = \dfrac{5}{8}$

This justifies the construction.

2. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\dfrac{2}{3}$ of the corresponding sides of the first triangle.

Sol. **Step 1**Draw a line segment AB = 4 cm. Taking point A as centre, draw an arc of 5 cm radius. Similarly, taking point B as its centre, draw an arc of 6 cm radius. These arcs will intersect each other at point C. Now, AC = 5 cm and BC = 6 cm and ΔABC is the required triangle.

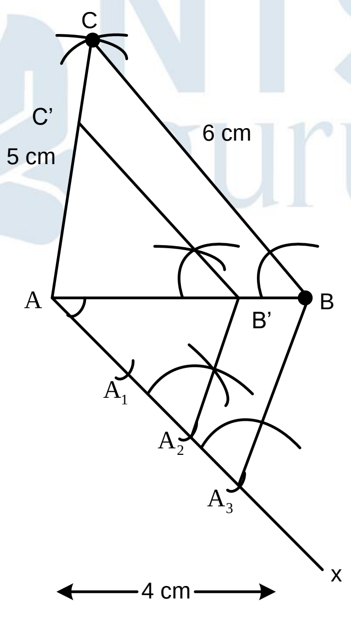
Step 2 Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.

Step 3 Locate 3 points A_1, A_2, A_3 (as 3 is greater between 2 and 3) on line AX such that $AA_1 = A_1A_2 = A_2A_3$

Step 4 Join BA_3 and draw a line through A_2 parallel to BA_3 to intersect AB at point B'.

Step 5 Draw a line through B' parallel to the line BC to intersect AC at C'

$\Delta AB'C'$ **is the required triangle.**



Justification

The construction can be justified by proving that

$AB' = \frac{2}{3}AB,\ B'C' = \frac{2}{3}BC, AC' = \frac{2}{3}\text{AC}$

By construction, we have $B'C' \parallel BC$

$\therefore \angle AB'C' = \angle ABC$ (Corresponding angles)

In $\Delta AB'C'$ and ΔABC,

$\angle AB'C' = \angle ABC$ (Proved above)

$\angle B'AC' = \angle BAC$ (Common)

$\therefore \Delta AB'C' \sim \Delta \text{ABC}$ (AA Similarity criterion)

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} \qquad ...(1)$$

In $\Delta AA_2B'$ and ΔAA_3B,

$\angle A_2AB' = \angle A_3AB$ (Common)

$\angle AA_2B' = \angle AA_3B$ (Corresponding angles)

$\therefore \Delta AA_2B' \sim \Delta AA_3B$ (AA similarity criterion)

$$\Rightarrow \frac{AB'}{AB} = \frac{AA_2}{AA_2}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{2}{3} \qquad ...(2)$$

From equation (1) and (2) we obtain

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{2}{3}$$

$$\Rightarrow AB' = \frac{2}{3}AB,\ B'C' = \frac{2}{3}BC, AC' = \frac{2}{3}AC$$

This justifies the construction.

3. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Sol. Let us assume that ΔABC is an isosceles triangle having CA and CB of equal lengths, base AB of 8 cm, and AD is the altitude of 4 cm.

$A \Delta AB'C'$ whose sides are $\frac{3}{2}$ times ΔABC can be drawn as follows.

Step 1 Draw a line segment AB of 8 cm. Draw arcs of same radius on both sides of the line segment while taking point A and B as its centre. Let these arcs intersect each other at O and O'. Join OO'. Let OO' intersect AB at D.

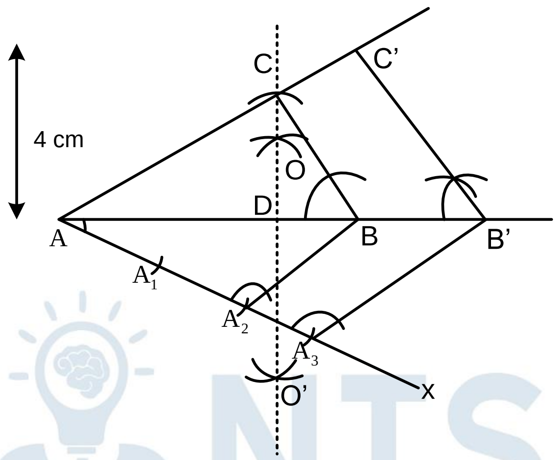
Step 2 Taking D as centre, draw an arc of 4 cm radius which cuts the extended line segment OO' at point C. An isosceles ΔABC is formed, having CD (altitude) as 4 cm and AB (base) as 8 cm.

Step 3 Draw a ray AX making an acute angle with line segment AB on the opposite side of vertex C.

Step 4 Locate 3 points (as 3 is greater between 3 and 2) $A_1, A_2,$ and A_3 on AX such that $AA_1 = A_1A_2 = A_2A_3$.

Step 5 Join BA_2 and draw a line through A_3 parallel to BA_3 to intersect extended line segment AB at point B'.

Step 6 Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. $\Delta AB'C'$ is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{3}{2}AB, B'C' = \frac{3}{2}BC, AC' = \frac{3}{2}AC$$

In ΔABC and $\Delta AB'C'$,

$\angle ABC = \angle AB'C'$ (Corresponding angles)

$\angle BAC = \angle B'AC'$ (Common)

$\therefore \Delta ABC \sim \Delta AB'C'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} \qquad ...(1)$$

In ΔAA_2B and $\Delta AA_3B'$,

$\angle A_2AB = \angle A_3AB'$ Common

$\angle AA_2B = \angle AA_3B'$ (Corresponding angles)

$\therefore \Delta AA_2B \sim \Delta AA_3B'$ (AA similarity criterion)

$\Rightarrow \dfrac{AB}{AB'} = \dfrac{AA_2}{AA_3}$

$\Rightarrow \dfrac{AB}{AB'} = \dfrac{2}{3}$ …(2)

On comparing equations (1) and (2), we obtain

$\dfrac{AB}{AB'} = \dfrac{BC}{B'C'} = \dfrac{AC}{AC'} = \dfrac{2}{3}$

$\Rightarrow AB' = \dfrac{3}{2}AB,\ B'C' = \dfrac{3}{2}BC, AC' = \dfrac{3}{2}AC$

This justifies the construction.

4. Draw a triangle ABC with side $BC = 6$ cm, $AB = 5$cm and $\angle ABC = 60^o$. Then construct a triangle whose sides are $\dfrac{3}{4}$ of the corresponding sides of the triangle ABC.

Sol. A $\Delta A'BC'$ whose sides are $\dfrac{3}{4}$ of the corresponding sides of ΔABC can be drawn as follows.

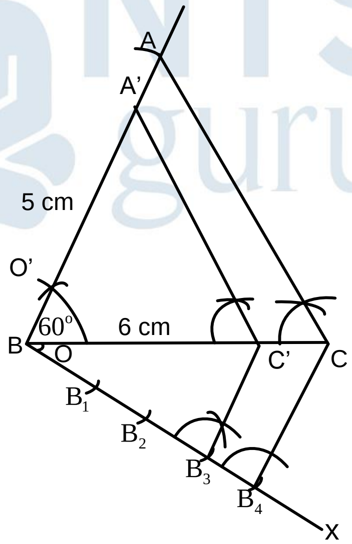
Step 1 Draw a ΔABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^o$.

Step 2 Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3 Locate 4 points (as 4 is greater in 3 and 4), B_1, B_2, B_3, B_4, on line segment BX.

Step 4 Join B_4C and draw a line through B_3, parallel to B_4C intersecting BC at C'.

Step 5 Draw a line through C' parallel to AC intersecting AB at A'. $\Delta A'BC'$ is the required triangle.



Justification

The construction can be justified by proving

$A'B=\frac{3}{4}AB, BC'=\frac{3}{4}BC, A'C'=\frac{3}{4}AC$

In $\Delta A'BC'$ and ΔABC,

$\angle A'C'B=\angle ACB$ (Corresponding angles)

$\angle A'BC'=\angle ABC$ (Common)

$\therefore \Delta A'BC' \sim \Delta ABC$ (AA similarity criterion)

$\Rightarrow \frac{A'B}{AB}=\frac{BC'}{BC}=\frac{A'C'}{AC}$ …(1)

In $\Delta BB_3C'$ and ΔBB_4C,

$\angle B_3BC'=\angle B_4BC$ (Common)

$\angle BB_3C'=\angle BB_4C$ (Corresponding angels)

$\therefore \Delta BB_3C' \sim \Delta BB_4C$ (AA similarity criterion)

$\Rightarrow \frac{BC'}{BC}=\frac{BB_3}{BB_4}$

$\Rightarrow \frac{BC'}{BC}=\frac{3}{4}$ …(2)

From equations (1) and (2), we obtain

$\frac{A'B}{AB}=\frac{BC'}{BC}=\frac{A'C'}{AC}=\frac{3}{4}$

$\Rightarrow A'B=\frac{3}{4}AB, BC'=\frac{3}{4}BC, A'C'=\frac{3}{4}AC$

This justifies the construction.

5. Draw a triangle ABC with side BC = 7 cm, $\angle B=45^o$, $\angle A=105^o$. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

Sol. $\angle B=45^o$, $\angle A=105^o$

Sum of all interior angles in a triangle is 180^o.

$\angle A+\angle B+\angle C=180^o$

$105^o+45^o+\angle C=180^o$

$\angle C=180^o-150^o$

$\angle C=30^o$

The required triangle can be drawn as follows.

Step 1 Draw a ΔABC with side BC = 7 cm, $\angle B = 45^o$, $\angle C = 30^o$

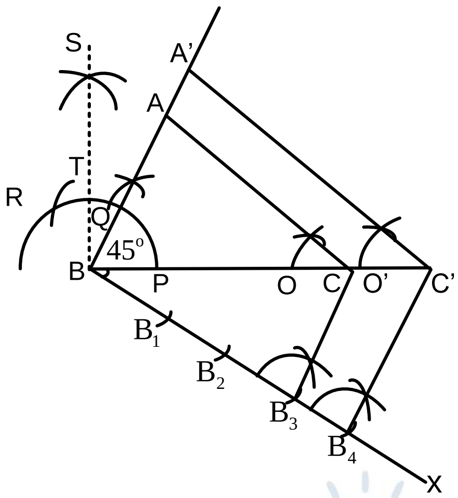
Step 2 Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3 Locate 4 Points (as 4 is greater in 4 and 3), $B_1, B_2, B_3, B_4,$ on BX.

Step 4 Join B_3C. Draw a line through B_4 parallel to B_3C intersecting extended BC at C'.

Step 5 Through C', draw a line parallel to AC intersecting extended line segment at C'.

$\Delta A'BC'$ is the required triangle.



Justification

The construction can be justified by proving that

$$A'B = \frac{4}{3}AB,\ BC' = \frac{4}{3}BC,\ A'C' = \frac{4}{3}AC$$

In ΔABC and $\Delta A'BC'$,

$\angle ABC = \angle A'BC'$ (Common)

$\angle ACB = \angle A'C'B$ (Corresponding angles)

$\therefore ABC \sim \Delta A'BC'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} \qquad \ldots(1)$$

In ΔBB_3C and $\Delta BB_4C'$,

$\angle B_3BC = \angle B_4B'$ (Common)

$\angle BB_3C = \angle BB_4C'$ (Corresponding angles)

$\therefore \Delta BB_3C \sim \Delta BB_4C'$ (AA similarity criterion)

$$\Rightarrow \frac{BC}{BC'} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC}{BC'} = \frac{3}{4} \qquad \ldots(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{A'B}=\frac{BC}{BC'}=\frac{AC}{A'C'}=\frac{3}{4}$$

$$\Rightarrow A'B=\frac{4}{3}AB, BC'=\frac{4}{3}BC, A'C'=\frac{4}{3}AC$$

This justifies the construction.

6. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

Sol. It is given that sides other than hypotenuse are of lengths 4 cm and 3 cm. Clearly, these will be perpendicular to each other.

The required triangle can be drawn as follows.

Step 1 Draw a line segment AB = 4 cm. Draw a ray SA making 90^{o} with it.

Step 2 Draw an arc of 3 cm radius while taking A as its centre to intersect SA at C. Join BC.

ΔABC **is the required triangle.**

Step 3 Draw a ray AX making an acute angle with AB, opposite to vertex C.

Step 4 Locate 5 points (as 5 is greater in 5 and 3), A_1, A_2, A_3, A_4, A_5, on line segment AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

Step 5 Join A_3B . Draw a line through A_5 parallel to A_3B intersecting extended line segment AB at B'.

Step 6

$$\Rightarrow \frac{AB}{AB'}=\frac{AA_3}{AA_5}$$

$$\Rightarrow \frac{AB}{AB'}=\frac{3}{5} \qquad \textbf{...(2)}$$

On comparing equations (1) and (2), we obtain

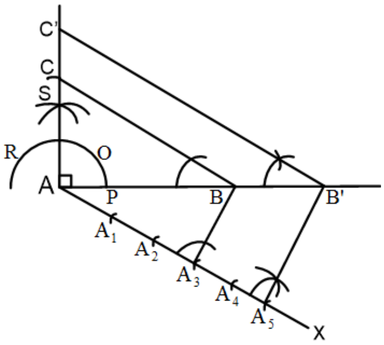
$$\frac{AB}{AB'}=\frac{BC}{B'C'}=\frac{AC}{AC'}=\frac{3}{5}$$

$$\Rightarrow AB'=\frac{5}{3}AB,\ B'C'=\frac{5}{3}BC,\ AC'=\frac{5}{3}AC$$

This justifies then construction.

Through B', draw a line parallel to BC intersecting extended line segment AC at C'.

$\Delta AB'C'$ is the required triangle.



Justification

The construction can be justified by proving that

$AB' = \frac{5}{3}AB,\ B'C' = \frac{5}{3}BC,\ AC' = \frac{5}{3}AC$

In ΔABC and $\Delta AB'C'$,

$\angle ABC = \angle AB'C'$ (Corresponding angles)

$\angle BAC = \angle B'AC'$ (Common)

$\therefore \Delta ABC \sim \Delta AB'C'$ (AA similarity criterion)

$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC}$ ...(1)

In ΔAA_3B and $\Delta AA_5B'$,

$\angle A_3AB = \angle A_5AB'$ (Common)

$\angle AA_3B = \angle AA_5B'$ (Corresponding angles)

$\therefore \Delta AA_3B \sim \Delta AA_5B'$ (AA similarity criterion)

7. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Sol. A pair of tangents to the given circle can be constructed as follows.

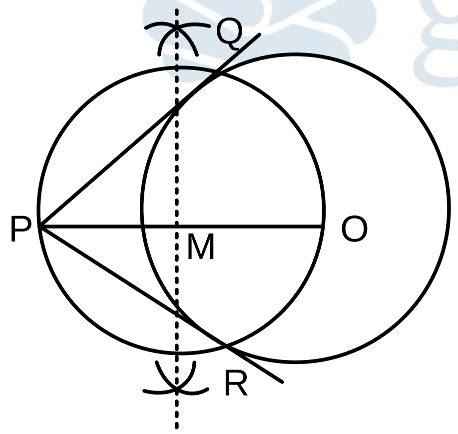
Step 1 Taking any point O of the given plane as centre, draw a circle of 6 cm radius. Locate a point P, 10 cm away from O. Join OP.

Step 2 Bisect OP. Let M be the mid-point of PO.

Step 3 Taking M as centre and MO as radius, draw a circle.

Step 4 Let this circle intersect the previous circle at point Q and R.

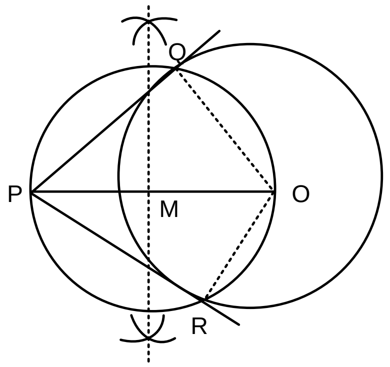
Step 5 Join PQ and PR. PQ and PR are the required tangents.



The lengths of tangents PQ and PR are 8 cm each.

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 6 cm). For this, join OQ and OR.



$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle PQO = 90^o$

$\Rightarrow OQ \perp PQ$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle.

8. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

Sol. Tangents on the given circle can be drawn as follows.

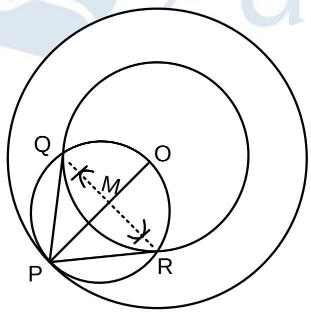
Step 1 Draw a circle of 4 cm radius with centre as O on the given plane.

Step 2 Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.

Step 3 Bisect OP. Let M be the mid-point of PO.

Step 4 Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at the points Q and R.

Step 5 Join PQ and PR. PQ and PR are the required tangents.



It can be observed that PQ and PR are of length 4.47 cm each.

In ΔPQO,

Since PQ is tangent,

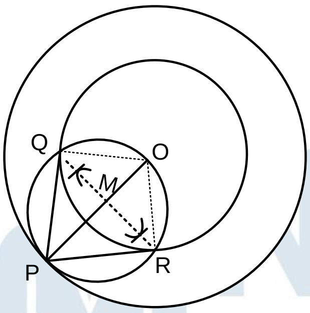
$\angle PQO = 90^o$

PO = 6 cm
QO = 4 cm
Appling Pythagoras theorem in ΔPQO, we obtain

$$PQ^2 + QO^2 = PQ^2$$
$$PQ^2 + (4)^2 = (6)^2$$
$$PQ^2 + 16 = 36$$
$$PQ^2 = 36 - 16$$
$$PQ^2 = 20$$
$$PQ = 2\sqrt{5}$$
$$PQ = 4.47\,cm$$

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 4 cm). For this, let us join OQ and OR.



$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle PQO = 90^{o}$

$\Rightarrow OQ \perp PQ$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle.

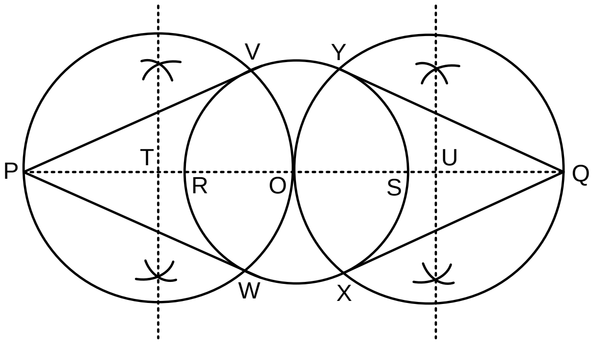
9. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

Sol. The tangent can be constructed on the given circle as follows.

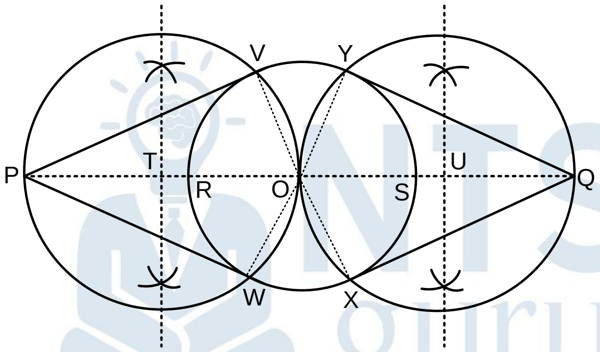
<u>Step 1</u>Taking any point O on the given plane as centre, draw a circle of 3 cm radius.

<u>Step 2</u>Take one of its diameters, RS, and extend it on both sides. Locate two points on this diameter such that OP = OS = 7 cm

Step 3 Bisect P and OQ. Let T and U be the mid-points of OP and OQ respectively.
Step 4 Taking T and U as its centre and with TO and UO as radius, draw two circles. These two circles will intersect the circle at point V, W, X, Y respectively. Join PV, PW, QX and QY. These are the required tangents.



Justification
The construction can be justified by proving that PV, PW, QY, and QX are the tangents to the circle (whose centre is O and radius is 3 cm). For this, join OV, OW, OX, and OV.



$\angle PVO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

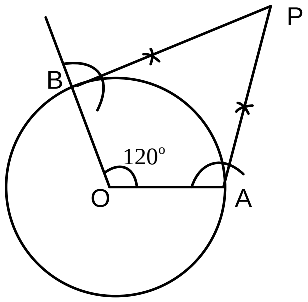
$\therefore \angle PVO = 90^{\circ}$

$\Rightarrow OV \perp PV$

Since OV is the radius of the circle, PV has to be a tangent of the circle. Similarly, PW, QX, and QY are the tangents of the circle.

10. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.
Sol. The tangents can be constructed in the following manner:
Step 1 Draw a circle of radius 5 cm and with centre as O.
Step 2 Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.
Step 3 Draw a radius OB, making an angle of $120^{\circ}(180^{\circ}-60^{\circ})$ with OA.

Step 4 Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P. PA and PB are the required tangents at an angle of 60°.



Justification

The construction can be justified by proving that $\angle APB = 60^{o}$

By our construction

$\angle OAP = 90^{o}$

$\angle OBP = 90^{o}$

And $\angle AOB = 120^{o}$

We know that the sum of all interior angles of a quadrilateral $= 360^{o}$

$\angle OAP + \angle AOB + \angle OBP + \angle APB = 360^{o}$

$90^{o} + 120^{o} + 90^{o} + \angle APB = 360^{o}$

$\angle APB = 60^{o}$

This justifies the construction.