Unless stated otherwise, use $\pi = \frac{22}{7}$

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Sol. Radius (r_1) of 1st circle = 19 cm

Radius (r_2) of 2nd circle = 9 cm

Let the radius of 3rd circle be r.

Circumference of 1st circle $= 2\pi r_1 = 2\pi(19) = 38\pi$

Circumference of 2nd circle $= 2\pi r_2 = 2\pi(9) = 18\pi$

Circumference of 3rd circle $= 2\pi r$

Given that,

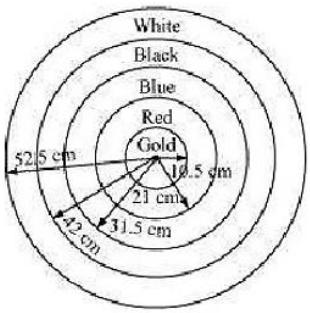
Circumference of 3rd circle = Circumference of 1st circle + Circumference of 2nd circle $2\pi r = 38\pi + 18\pi = 56\pi$

$r = \frac{56\pi}{2\pi} = 28$

Therefore, the radius of the circle which has circumference equal to the sum of the circumference of the given two circles is 28 cm.

2. Given Fig. Depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Sol.



Radius (r_1) of gold region (i.e., 1st circle) $= \frac{21}{2} = 10.5$ cm

Given that each circle is 10.5 cm wider than the previous circle.

Therefore, radius (r_2) of 2nd circle = 10.5 + 10.5

= 21 cm

Radius (r_3) of 3rd circle = 21 + 10.5

= 31.5 cm

Radius (r_4) of 4th circle = 31.5 + 10.5

= 42

Radius (r_5) of 5th circle = 42 + 10.5

= 52.5 cm

Area of gold region = Area of 1st circle $= \pi r_1^2 = \pi(10.5)^2 = 346.5 \text{ cm}^2$

Area of red region = Area of 2nd circle – Area of 1st circle

$= \pi r_2^2 - \pi r_1^2$

$= \pi(21)^2 - \pi(10.5)^2$

$= 441\pi - 110.25\pi = 330.75\pi$

$= 1039.5 cm^2$

Area of blue region = Area of 3rd circle – Area of 2nd circle.

$= \pi r_3^2 - \pi r_2^2$

$= \pi(31.5)^2 - \pi(21)^2$

$= 992.25\pi - 441\pi = 551.25\pi$

$= 1732.5 cm^2$

Area of black region = Area of 4th circle – Area of 3rd circle

$= \pi r_4^2 - \pi r_3^2$

$= \pi(42)^2 - \pi(31.5)^2$

$= 1764\pi - 992.25\pi$

$= 771.75\pi = 2425.5 cm^2$

Area of white region = Area of 5th circle – Area of 4th circle

$= \pi r_5^2 - \pi r_4^2$

$= \pi(52.5)^2 - \pi(42)^2$

$= 2756.25\pi - 1764\pi$

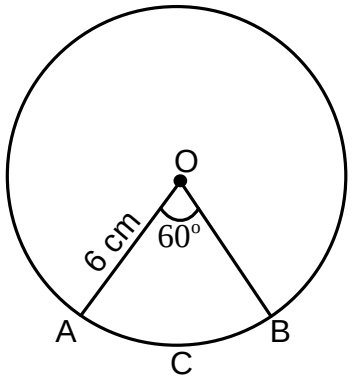
$= 992.25\pi = 3118.5 cm^2$

Therefore, areas of gold, red, blue, black, and white regions are $346.5 cm^2, 1039.5\ cm^2, 1732.5 cm^2, 2425.5 cm^2$, and $3118.5 cm^2$ respectively.

Unless stated otherwise, use $\pi = \frac{22}{7}$.

3. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Sol.



Let OACB be a sector of the circle making 60^o angle at centre O of the circle.

Area of sector of angle $\theta = \frac{\theta}{360^o} \times \pi r^2$

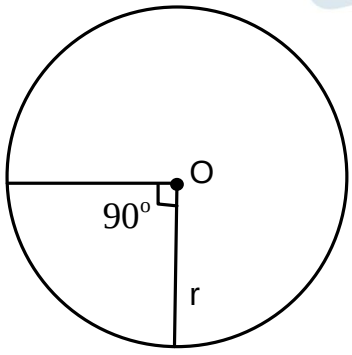
Area of sector $OACB = \frac{60^o}{360^o} \times \frac{22}{7} \times (6)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{cm}^2$

Therefore, the area of the sector of the circle making 60^o at the centre of the circle is $\frac{132}{7} \text{cm}^2$

4. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol.



Let the radius of the circle be r.

Circumference = 22 cm

$2\pi r = 22$

$r = \frac{22}{2\pi} = \frac{11}{\pi}$

Quadrant of circle will subtend 90^o angle at the centre of the circle.

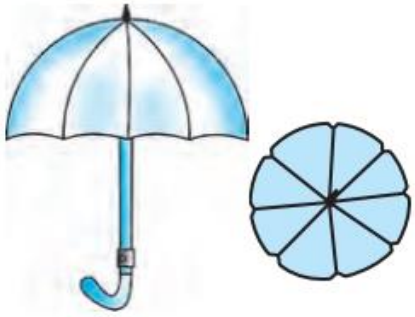
Area of such quadrant of the circle $= \frac{90^o}{360^o} \times \pi \times r^2$

$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2 = \frac{1}{4} \times \pi \times \frac{121}{\pi^2}$

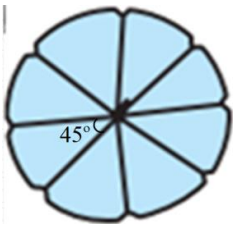
$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$

$= \frac{77}{8} \text{cm}^2$

5. An umbrella has 8 ribs which are equally spaced (see Fig.). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Sol. There are 8 ribs in an umbrella. The area between two consecutive ribs is subtending $\frac{360^o}{8} = 45^o$ at the centre of the assumed flat circle.

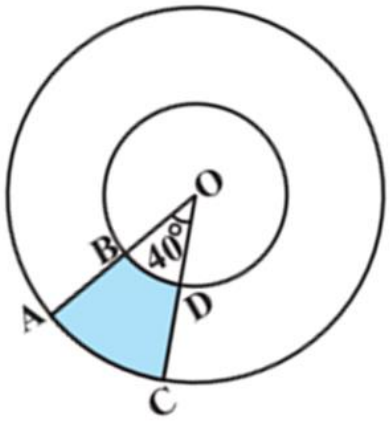


Area between two consecutive ribs of circle $= \frac{45^o}{360^o} \times \pi r^2$

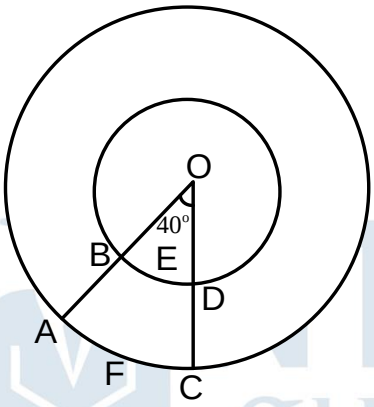
$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$

$= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{cm}^2.$

6. Find the area of the shaded region in the. Given fig. If radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^o$.



Sol.



Radius of inner circle = 7 cm
Radius of outer circle = 14 cm
Area of shaded region = Area of sector OAFC – Area of sector OBED

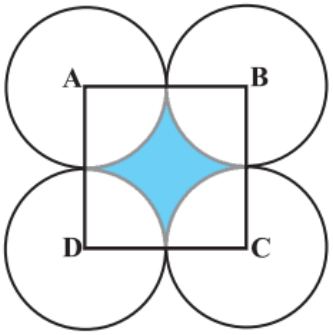
$$= \frac{40^o}{360^o} \times \pi(14)^2 = \frac{40^o}{360^o} \times \pi(7)^2$$

$$= \frac{1}{9} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{9} \times \frac{22}{7} \times 7 \times 7$$

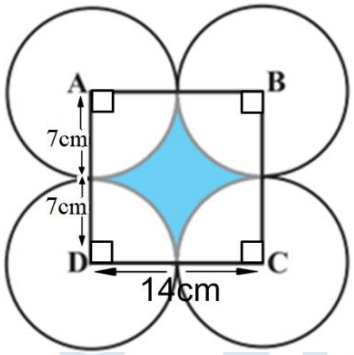
$$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9}$$

$$= \frac{154}{3} cm^2.$$

7. In the given figure. ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



Sol.



Area of each of the 4 sectors is equal to each other and is a sector of 90^o in a circle of 7 cm radius.

Area of each sector $= \frac{90^o}{360^o} \times \pi (7)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

$= \frac{77}{2} \text{cm}^2$

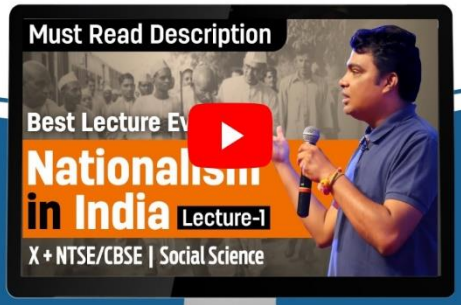
Area of square $ABCD = (\text{Side})^2 = (14)^2 = 196 \text{ cm}^2$

Area of shaded portion = Area of square $ABCD - 4 \times$ Area of each sector

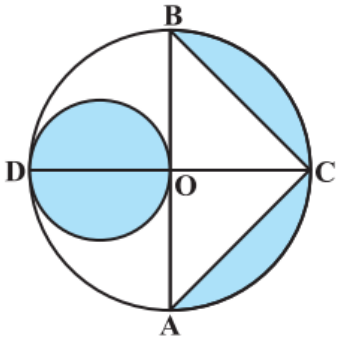
$= 196 - 4 \times \frac{77}{2} = 196 - 154$

$= 42 \text{cm}^2$

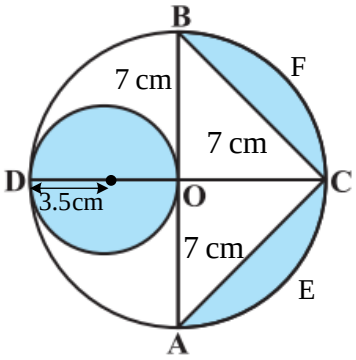
Therefore, the area of shaded portion is 42 cm^2.



8. In the given figure. AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



Sol.



Radius (r_1) of larger circle = 7 cm

Radius (r_2) of smaller circle $= \frac{7}{2}$ cm

Area of smaller circle $= \pi r_1^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2} \text{cm}^2$

Area of semi-circle AECFB of larger circle $= \frac{1}{2}\pi r_2^2$

$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$

$= 77 \text{cm}^2$

Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 14 \times 7 = 49 \text{cm}^2$

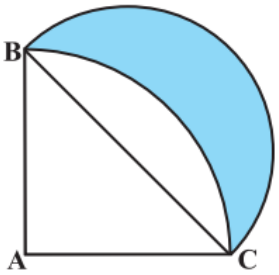
Area of the shaded region

= Area of smaller circle + Area of semi-circle AECFB – Area of ΔABC

$= \frac{77}{2} + 77 - 49$

$= 28 + \frac{77}{2} = 28 + 38.5 = 66.5 \text{ cm}^2$

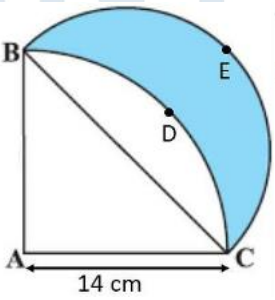
9. In given figure. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Sol. As ABC is a quadrant of the circle, $\angle BAC$ will be of measure 90^o.

In ΔABC,

$BC^2 = AC^2 + AB^2$



$= (14)^2 + (14)^2$

$BC = 14\sqrt{2}$

Radius (r_1) of semi-circle drawn on $BC = \frac{14\sqrt{2}}{2} = 7\sqrt{2}\, cm$

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 14 \times 14$

$= 98 \text{ cm}^2$

Area of sector $ABDC = \frac{90^o}{360^o} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154 \text{cm}^2$

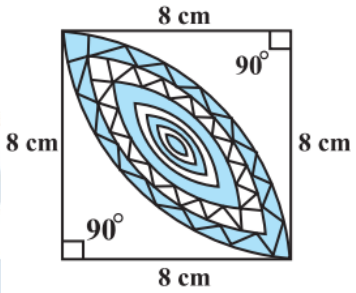
Area of semi-circle drawn on $BC = \frac{1}{2} \times \pi \times r_1^2 = \frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2 = \frac{1}{2} \times \frac{22}{7} \times 98 = 154 \text{ cm}^2$

Area of shaded region = Area of semi-circle – (Area of sector ABDC –Area of ΔABC)

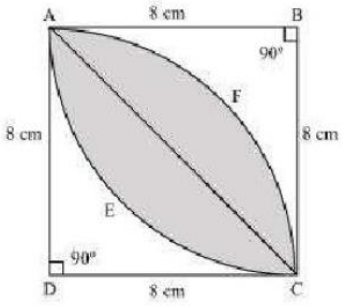
$= 154 - (154 - 98)$

$= 98 \text{ cm}^2$

10. Calculate the area of the designed region in the given figure common between the two quadrants of circles of radius 8 cm each.



Sol.



The designed area is the common region between two sectors BAEC and DAFC.

Area of sector $BAEC = \frac{90^o}{360^o} \times \frac{22}{7} \times (8)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 64$

$= \frac{22 \times 16}{7}$

$= \frac{352}{7} \text{cm}^2$

Area of $\Delta BAC = \frac{1}{2} \times BA \times BC$

$= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2$

Area of the designed portion $= 2 \times$ (Area of segment AEC)

$= 2 \times$ (Area of sector BAEC – Area of ΔBAC)

$= 2 \times \left(\frac{352}{7} - 32 \right) = 2 \left(\frac{352 - 224}{7} \right)$

$= \frac{2 \times 128}{7}$

$= \frac{256}{7} cm^2$