1. Find the distance between the following pairs of points :
(i) (2, 3), (4, 1) (ii) (– 5, 7), (– 1, 3) (iii) (a, b), (– a, – b)

Sol. **(i)** Let the given points be $P(2,3)$ and $Q(4,1)$.

Then $x_1 = 2, y_1 = 3, x_2 = 4$ and $y_2 = 1$

$\therefore$ Distance $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-2)^2 + (1-3)^2}$

$= \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ Units.

(ii) Let the given points be $P(-5,7)$ and $Q(-1,3)$.

Then $x_1 = -5, y_1 = 7, x_2 = -1$ and $y_2 = 3$

$\therefore$ Distance $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1+5)^2 + (3-7)^2}$

$= \sqrt{(4)^2 + (-4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ Units.

(iii) Let the given points be $P(a,b)$ and $Q(-a,-b)$.

Then $x_1 = a, y_1 = b, x_2 = -a$ and $y_2 = -b$

$\therefore$ Distance $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-a-a)^2 + (-b-b)^2}$

$= \sqrt{(-2a)^2 + (-2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= 2\sqrt{a^2 + b^2}$ Units.

2. Find the distance between the points (0, 0) and (36, 15).

Sol. Let points be $A(0,0)$ and $B(36,15)$

The distance between two points is

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(36-0)^2+(15-0)^2}$$

$$=\sqrt{1296+225}=\sqrt{1521}=39 \text{ units}$$

3. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Sol. Let points be $A(5,-2), B(6,4)$ and $C(7,-2)$

$$AB=\sqrt{(6-5)^2+(4+2)^2}=\sqrt{1+36}=\sqrt{37}$$

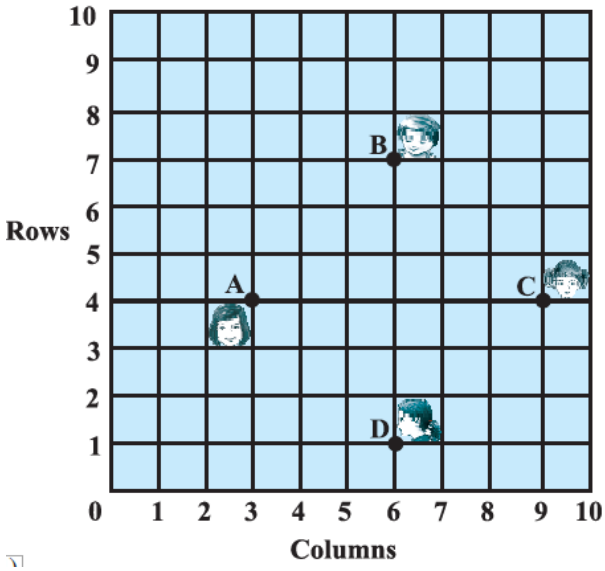
$$BC=\sqrt{(7-6)^2+(-2-4)^2}=\sqrt{1+36}=\sqrt{37}$$

$$AC=\sqrt{(7-5)^2+(-2+2)^2}=\sqrt{4+0}=2$$

Here, $AB=BC$

ΔABC is an isosceles triangle.

4. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Sol. From the figure, let the points along with coordinates be $A(3,4), B(6,7), C(9,4)$ and $D(6,1)$.

Then by distance formula, we have

$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{18} = 3\sqrt{2}.$

$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{18} = 3\sqrt{2}.$

$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{18} = 3\sqrt{2}.$

$DA = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{18} = 3\sqrt{2}.$

Also, diagonal $AC = \sqrt{(9-3)^2 + (4-4)^2}$

$= \sqrt{(6)^2 + (0)^2} = \sqrt{36} = 6.$

and diagonal $BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{(0)^2 + (-6)} = \sqrt{36} = 6.$

$\therefore$ $AB = BC = CD = DA = 3\sqrt{2}$ and diagonals $AC = BD = 6$

Thus, ABCD is a square and Champa is correct.

5. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).

Sol. Let $A(2,-5)$ and $B(-2,9)$ be the given points.

Also let $P(x,0)$ be the point on x-axis such that

$PA = PB$

Then $PA^2 = PB^2$

$\Rightarrow \quad (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$

$\Rightarrow \quad (x-2)^2 - (x+2)^2 = 81 - 25$

$\Rightarrow \quad (x-2+x+2)(x-2-x-2) = 56$

$\Rightarrow \quad (2x)(-4) = 56$

$\Rightarrow \quad -8x = 56$

$\Rightarrow \quad x = -7$

Hence, the required point is $(-7,0)$.

6. Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.

Sol. Points $P(2,-3), Q(10,y)$ and $PQ=10$ units

The distance between two points is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=PQ \Rightarrow \sqrt{(10-2)^2+(y+3)^2}=10$$

$$\Rightarrow \quad 64+y^2+9+6y=100 \Rightarrow y^2+6y+73-100=0$$

$$\Rightarrow \quad y^2+6y-27=0 \Rightarrow y^2+9y-3y-27=0$$

$$\Rightarrow \quad y(y+9)-3(y+9)=0 \Rightarrow (y-3)(y+9)=0$$

$$\Rightarrow \quad y-3=0 \text{ or } y+9=0$$

$$\Rightarrow \quad y=3 \text{ or } -9$$

7. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

Sol. Let the coordinates of point C be (x, y).

$$x-\text{coordinate of } C=\frac{mx_2+nx_1}{m+n}=\frac{2\times 4+3\times(-1)}{2+3}=\frac{8-3}{5}=1.$$

$$y-\text{coordinate of } C=\frac{my_2+ny_1}{m+n}=\frac{2\times(-3)+3\times(7)}{2+3}=\frac{-6+21}{5}=3.$$

Hence, the coordinates of C are $(1,3)$.

8. Find the area of the triangle whose vertices are :

(i) **(2, 3), (–1, 0), (2, – 4)**

(ii) **(–5, –1), (3, –5), (5, 2)**

Sol. **(i)** Here, $x_1=2, y_1=3, x_2=-1, y_2=0, x_3=2$ and $y_3=-4$

$\therefore$ Area of triangle

$$=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$=\frac{1}{2}\left[\{2(0+4)\}+\{(-1)(-4-3)\}+\{2(3-0)\}\right]$$

$$=\frac{1}{2}[8+7+6]=\frac{21}{2}=10.5 \text{ sq units}.$$

(ii) Area of triangle

Here, $x_1 = -5$, $y_1 = -1$, $x_2 = 3$, $y_2 = -5$, $x_3 = 5$, $y_3 = 2$

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[\{-5(-5-2)\} + \{3(2+1)\} + \{5(-1+5)\}\right]$$

$$= \frac{1}{2}[35 + 9 + 20] = \frac{1}{2} \times 64 = 32 \text{ sq units}.$$

9. In each of the following find the value of 'k', for which the points are collinear.

(i) (7, –2), (5, 1), (3, k)

(ii) (8, 1), (k, – 4), (2, –5)

Sol. **(i)** Points $A\overset{x_1}{(7}, \overset{y_1}{-2}), B\overset{x_2}{(5}, \overset{y_2}{1}), C\overset{x_3}{(3}, \overset{y_3}{k})$

For collinear points,

Area of $\Delta ABC = 0$

Area of $\Delta ABC = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$$\Rightarrow \quad \frac{1}{2}\left[7(1-k) + 5(k+2) + 3(-2-1)\right] = 0$$

$$\Rightarrow \quad 7 - 7k + 5k + 10 - 9 = 0$$

$$\Rightarrow \quad k = \frac{8}{2}$$

$$\Rightarrow \quad k = 4$$

(ii) $A(8,1), B(k,-4), C(2,-5)$

For collinear points, Area of $\Delta ABC = 0$

$$\Rightarrow \quad \frac{1}{2}\left[8(-4+5) + k(-5-1) + 2(1+4)\right] = 0$$

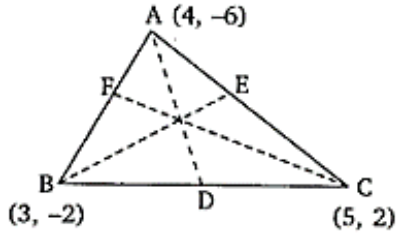
$$\Rightarrow \quad 8 - 6k + 10 = 0$$

$$\Rightarrow \quad 6k = 18$$

$$\Rightarrow \quad k = 3$$

10. You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A(4, – 6), B(3, –2) and C(5, 2).

Sol. Let AD be the median of ΔABC.



Then

Coordinates of mid-point $D = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$

$\therefore$ Area of triangle

$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Area of $\Delta ABD = \frac{1}{2}\left[4(-2-0) + 3(0+6) + 4(-6+2)\right]$

$= \frac{1}{2}[-8 + 18 - 16]$

$= \frac{1}{2}[-6]$

$= -3 \text{ sq units.}$ **.....(i)**

Area of ΔADC

$= \frac{1}{2}\left[4(0-2) + 4(2+6) + 5(-6-0)\right]$

$= \frac{1}{2}[-8 + 32 - 30]$

$= \frac{1}{2}[-6]$

$= -3 \text{ sq units.}$ .....(ii)

From equations (i) and (ii), we have:

Area of ΔABD = Area of ΔADC

Similarly, we can show for the other medians. Hence, verified.