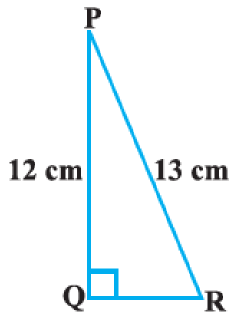
NCERT Solutions for Class 10
MATHS – Introduction To Trigonometry

1. In Figure, find tan P – cot R.



Sol. **In right angled ΔPQR,**

$$PR^2 = PQ^2 + QR^2 \quad \textbf{[Pythagoras Theorem]}$$

$\Rightarrow \quad (13)^2 = (12)^2 + QR^2$

$\Rightarrow \quad 169 - 144 = QR^2$

$\Rightarrow \quad 25 = QR^2$

$\Rightarrow \quad QR = 5cm$

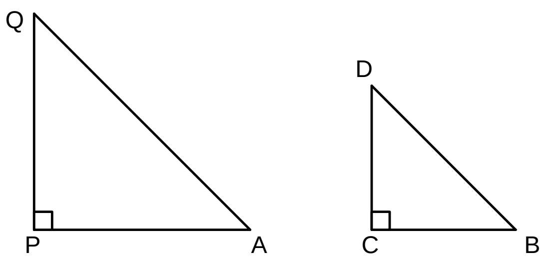
$$\tan P = \frac{QR}{PQ} = \frac{5}{12}$$

$$\cot R = \frac{QR}{PQ} = \frac{5}{12}$$

So, $\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

2. If ∠ A and ∠ B are acute angles such that cos A = cos B, then show that ∠ A = ∠ B.

Sol. Given that $\cos A = \cos B$



$$\Rightarrow \frac{AP}{AQ} = \frac{BC}{BD}$$

$$\Rightarrow \quad \frac{AP}{BC} = \frac{AQ}{BD} = \text{k} \qquad \ldots\ldots\text{(i)}$$

Let $AP = k(BC)$ and $AQ = k(BD)$

Now in ΔAPQ and ΔBCD

$$\frac{PQ}{CD} = \frac{\sqrt{AQ^2 - AP^2}}{\sqrt{BD^2 - BC^2}} = \frac{\sqrt{(k.BD)^2 - (k.BC)^2}}{\sqrt{BD^2 - BC^2}}$$

$$= \frac{k\sqrt{BD^2 - BC^2}}{\sqrt{BD^2 - BC^2}} = k \qquad \ldots\ldots\text{..(ii)}$$

From equation (i) and (ii) we get

$$\frac{AP}{BC} = \frac{AQ}{BD} = \frac{PQ}{CD} \qquad \text{So, } \Delta APQ \sim \Delta BCD$$

Hence $\angle A = \angle B$

3. If 3 cot A = 4, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 \text{A} - \sin^2 \text{A}$ or not.

Sol. Given that : $3\cot A = 4 \qquad \Rightarrow \cot A = \frac{4}{3}$

Let $\cot A = \frac{4k}{3k}$, where k is any non-zero real number.

In ΔABC, by Pythagoras theorem we have

$$AC^2 = AB^2 + BC^2$$
$$= (3k)^2 + (4k)^2$$
$$= 25k^2$$
$$AC = 5k$$

Therefore,

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{7}{25}$$

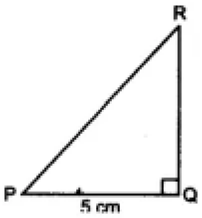
and

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Hence, $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

4. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Sol. In right angled ΔPQR



$$PR^2 = PQ^2 + QR^2$$

$\Rightarrow \quad (5)^2 = (PR+QR)(PR-QR)$

$\Rightarrow \quad 25 = 25(PR+QR)(PR-QR)$

$\Rightarrow \quad PR - QR = 1$ ....(i)

and $\quad PR + QR = 25$ ......(ii)

On adding equation (i) and (ii), we get

$$2PR = 26 \quad \Rightarrow \quad PR = \frac{26}{2} = 13\,cm$$

From equation (i),

$$PR - QR = 1 \quad \Rightarrow \quad QR = 13 - 1$$

$$QR = 12\,cm$$

$$\sin P = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{QR}{PQ} = \frac{12}{5}$$

5. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) $\sec\theta = \frac{4}{3}$ for some angle θ.

Sol. **(i)** False, because $\tan 60° = \sqrt{3} > 1$. Also if opposite side > adjacent side then the value is greater then 1.

(ii) True, because $\sec A \geq 1$. Also hypotenuse is always greater than adjacent side.

(iii) False, because cos A abbreviation is used for cosine A.

(iv) False, because the term cot A is single not a product. cot A is used for cotangent of angle A.

(v) False, because $-1 \leq \sin\theta \leq 1$.

6. If $\tan 2\text{A} = \cot(\text{A}-18°)$, where 2A is an acute angle, find the value of A.

Sol. **We have** $\tan 2A = \cot(A-18°)$

$\Rightarrow \cot(90° - 2A) = \cot(A-18°)$ $\quad [\because \quad \cot(90° - \theta) = \tan\theta]$

$\Rightarrow 90° - 2A = A - 18°$

$\Rightarrow 2A + A = 90° + 18°$

$\Rightarrow 3A = 108°$

$\Rightarrow A = \frac{108}{3} = 36°$

7. If tan A = cot B, prove that $\text{A} + \text{B} = 90°$.

Sol. $\tan A = \cot B$

$\Rightarrow \tan A = \tan(90° - B)$ $\quad [\because \quad \tan(90° - \theta) = \cot\theta]$

$\Rightarrow A = 90° - B$

$\Rightarrow A + B = 90°$

Proved.

8. If $\sec 4\text{A} = \text{cosec}(A - 20°)$, where 4A is an acute angle, find the value of A.

Sol. **We have** $\sec 4A = \text{cosec}(A - 20°)$

$\Rightarrow \text{cosec}(90° - 4A) = \text{cosec}(A - 20°)$ $\quad [\because \quad \text{cosec}(90° - \theta) = \sec\theta]$

$\Rightarrow 90° - 4A = A - 20°$ $\quad \Rightarrow \quad 90° + 20° = 5A$

$\Rightarrow 110° = 5A$

$\Rightarrow \frac{110}{5} = A$

$\Rightarrow A = 22°$

9. Evaluate:

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Sol. **(i)** $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$ $\left[\because \cos(90° - \theta) = \sin\theta \text{ and } \sin(90° - \theta) = \cos\theta\right]$

$= \dfrac{\sin^2 63° + \sin^2(90° - 63°)}{\cos^2(90° - 73°) + \cos^2 73°}$

$= \dfrac{\sin^2 63° + \cos^2 63°}{\cos^2 73° + \cos^2 73°} = 1.$ $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

$= \sin 25° \cos(90° - 25°) + \cos 25° \sin(90° - 25°)$

$= \sin^2 25° + \cos^2 25° = 1.$

10. Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

(A) 0 (B) 1 (C) 2 (D) -1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) sec A (B) sin A (C) cosec A (D) cos A

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

Sol. **(i)** $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A) = 9 \times 1 = 9$ **Correct option is (B)**

$\left[\because \sec^2 A - \tan^2 A = 1\right]$

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = \left(\dfrac{1}{1} + \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)\left(\dfrac{1}{1} + \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)$

$= \left(\dfrac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\dfrac{\sin\theta + \cos\theta - 1}{\sin\theta}\right) = \dfrac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta \sin\theta}$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \dfrac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} = \dfrac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} = \dfrac{2\cos\theta\sin\theta}{\cos\theta\sin\theta} = 2$

Correct Option is (C)

(iii) $\left(\sec A + \tan A\right)\left(1-\sin A\right) = \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)\left(\frac{1-\sin A}{1}\right) = \left(\frac{1+\sin A}{\cos A}\right)\left(\frac{1-\sin A}{1}\right)$

$$= \frac{(1)^2 - (\sin A)^2}{\cos A} = \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A \qquad \left[\because 1-\sin^2 A = \cos^2 A\right]$$

Correct option is (D)

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A$

$$\left[\because 1+\tan^2 A = \sec^2 A \text{ and } 1+\cot^2 A = \operatorname{cosec}^2 A\right]$$